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ABSTRACT

This programed instruction study guide is one of a series that form a first-year algebra course. Structured in a multiple-choice question-answer format with scrambled pages, it is intended to be used in conjunction with a computer-managed instructional system. The following topics are covered in Volume 5: operations on directed numbers, transforming equations, and properties of inequalities. Reading and homework assignments are taken from the text "Modern Algebra - Book I" by Dolciani. (Related documents are SE 015 854 - SE 015 870.) (DT)

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PROGRAMMED MATH CONTINUUM

level one

ALGEBRA

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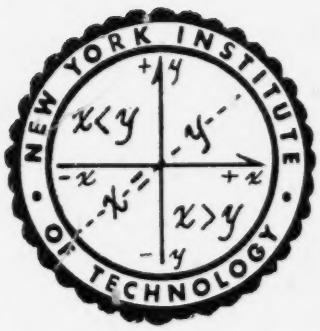

VOLUME

5

SE 015 860

NEW YORK INSTITUTE OF TECHNOLOGY
OLD WESTBURY, NEW YORK

ED 075206

PROGRAMMED MATH CONTINUUM

LEVEL ONE

A L G E B R A

VOLUME 5

New York Institute of Technology

Old Westbury - New York

PREFACE

This volume is one of a set of 18

that form a complete course

in

ALGEBRA - LEVEL ONE

The volume has been structured

in a multiple choice question-answer format,

with the pagination scrambled

and

is to be used in conjunction with

a program control console

utilizing

punch card input.

It is one exhibit in the demonstration of a model

developed under the direction of

the U.S. Department of Health Education and Welfare

Project 8-0157

at the

New York Institute of Technology

Westbury, New York

VOLUME 5

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IN THE STUDY GUIDE:

QUESTION:	SEGMENT:	IS ON PAGE:
1	1	$\frac{1}{1}$
1	2	$\frac{42}{1}$
1	3	$\frac{83}{1}$
1		$\frac{135}{1}$
1	5	$\frac{168}{1}$

VOLUME 5

This volume covers the following material as shown in this excerpt from the Syllabus:

SEGMENT	DESCRIPTION	REFERENCE BOOK SECTION		
		DOLCIANI	DRESSLER	DODES
1	Adding directed numbers	4-6	6- 7	3-6 , 3- 7
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2	Multiplying directed numbers	4-8	6- 8	3-9 , 3-10
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4	Transforming equations	5-1	9- 5	5-2 , 5-3
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READING ASSIGNMENT

VOLUME 5

Before you begin to answer the questions in this STUDY GUIDE you should read the pages indicated.

SEGMENT	FROM PAGE	TO PAGE
1	124	130
2	133	135
3	138	140
4	157	158
5	159	162

Modern Algebra Book I
Dolciani, Berman and Freilich
Houghton Mifflin, 1965

Read EVERYTHING contained in these pages.

EXAMINE every illustrative problem

Write in your NOTEBOOK:

1) Every RULE that has been stated
2) Every DEFINITION that has been presented
3) Solve at least ONE PROBLEM of each type covered in the lesson.

If you wish additional information for enrichment purposes consult:

Algebra I
Dodes and Greitzer
Hayden Book Co., 1967

You will be given additional notes at various places in the STUDY GUIDE. These, too, should be entered in your NOTEBOOK.

E

HOMEWORK ASSIGNMENT

VOLUME NO. 5

BOOK: DOLCIANI

HOMEWORK QUESTION NO.	PAGE NO.	EXAMPLE NUMBER	MBO REFERENCE
1	127	7 , 9 , 11 , 12	05110
2	127	13 , 14 , 15 , 18	05110
3	131	1 , 3 , 5 , 7	05120
4	131	14 , 17 , 19 , 20	05120
5	137	1 , 6 , 11 , 15	05210
6	137	21 , 26 , 32 , 37	05110 , 05120
7	138	49 , 50 , 52 , 53	05110 , 05120
8	138	54 , 55 , 56 , 57	05110 , 05210
9	141	1 , 3 , 5 , 7 , 9	05310
10	141	11 , 13 , 15 , 17	05310
11	141	19 , 21 , 23 , 25	05310
12	142	31 , 32 , 33	05310
13	159	2 , 4 , 6 , 8	05410 , 05420
14	159	10 , 12 , 17 , 19	05410 , 05420
15	159	13 , 21 , 24 , 25	05430
16	159	40 , 42 , 46	05410 , 05420
17	163	1 , 3 , 5 , 7	05531
18	163	11 , 13 , 15 , 17	05531
19	163	19 , 21 , 23 , 25	05531
20	163	29 , 30 , 32	05531

GENERAL INSTRUCTIONS

Ask your teacher for:

PUNCH CARD
PROGRAM CONTROL
ANSWER MATRIX

When you are ready at the PROGRAM CONTROL

Insert the PUNCH CARD in the holder
Turn to the first page of the STUDY GUIDE
Read all of the instructions
Read the First Question

Copy the question
Do your work in your notebook
Do all of the computation necessary
Read all of the answer choices given

Choose the Correct answer
(remember, once you've punched the card
it can't be changed)

Punch the card with the STYLUS

Read the instruction on the PROGRAM CONTROL
(it tells you which page to turn to)

TURN TO THAT PAGE:

If your choice is <u>not correct</u> you will be given additional hints, and will be directed to return to the question and to choose another answer.

If your choice is <u>correct</u> then you will be directed to proceed to the next question located immediately below, on the same page.

If you have no questions to ask your teacher now, you can turn the page and begin. If you have already completed a SEGMENT turn to the beginning of the following segment;

CHECK THE PAGE NUMBER BY LOOKING AT THE TABLE OF CONTENTS

VOLUME 5 SEGMENT 1 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 1	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 5	(Volume Number)
	66	and	68	0 1	(Segment Number)

Your READING ASSIGNMENT for this Segment is pages 124 - 130 .

SUPPLEMENTARY NOTES:

In the previous Volume, you have seen how the NUMBER LINE can be used as a reference to allow you to add and subtract signed numbers.

You will no doubt agree that although it gives a concrete illustration of the operation, it is a time consuming process. Now, just as you long ago advanced from counting on your fingers to the memorizing of the number combinations, you will now be told to advance from counting along the NUMBER LINE to memorizing the RULES FOR THE ADDITION AND SUBTRACTION OF SIGNED NUMBERS.

Of course, you will always be able to fall back, at least mentally, on the picture of the NUMBER LINE to support your memory; but your main task now is to commit the following rules to memory.

Please turn to page 2.

V

Statement or RULE:	Symbolic Form of RULE:
(1) The SUM of two positive numbers is positive. The absolute value of the sum is the sum of their absolute values.	(1) If $a > 0$ and $b > 0$ then, $a + b = +(\|a\| + \|b\|)$
(2) The SUM of two NEGATIVE numbers is negative. The absolute value of the sum is the sum of their absolute values.	(2) If $a < 0$ and $b < 0$ then, $a + b = -(\|a\| + \|b\|)$
(3) & (4) The SUM of a POSITIVE and a NEGATIVE number has the sign of the number with the larger absolute value. The absolute value of the sum is the DIFFERENCE of their absolute values.	(3) If $a > 0$ and $b < 0$ and $\|a\| > \|b\|$ then, $a + b = +(\|a\| - \|b\|)$ (4) If $a > 0$ and $b < 0$ and $\|b\| > \|a\|$ then, $a + b = -(\|b\| - \|a\|)$

RULE FOR SUBTRACTION:

Since subtraction can be accomplished by adding the " additive inverse " we make that substitution and then use the addition rule that applies.

$$a - (+b) = a + (-b)$$

$$a - (-b) = a + (+b)$$

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Apply the proper rule to find the value of

$$(+2) + (+7)$$

(A) 14

(B) + 9

(C) + 5

(D) - 5

$\frac{3}{1}$

Since the two negative quantities in parentheses are separated by a plus sign, this is an addition of two negative numbers.
Then Rule 2 should be applied.
Please review Rule 2 , since you have made a mistake.
Remember, most decisions in Algebra involve

(1) Recognizing which rule to use, and then

(2) Using the rule precisely.

Please return to page $\frac{19}{2}$ and try question 4 again.

$\frac{3}{2}$

Let us consider the inequality in two parts.
The left side asks for the absolute value of the sum of positive 7 and negative 8 . By Rule 4 , the sum of +7 and -8 is -1 . By the definition of absolute value, the absolute value of -1 is 1 .

The right side of the inequality asks for the sum of the individual absolute values. The absolute value of 7 is 7 ; the absolute value of negative 8 is 8 . The sum of these two absolute values by Rule 1 is 15 .

The Third step to take is to compare the left side with the right side.

Is the left side greater than the right side as the problem states? Is 1 greater than 15 ? Since this is not the case, then the inequality is not true.

Please return to page $\frac{29}{2}$ and reconsider this question.

V

$\frac{4}{1}$

It appears that you have added 7 and 10 , but the only time when it is proper to add absolute values is when you are adding numbers which have the same sign. That is not the case here.

Did you apply a subtraction rule? You should not have done so, because this problem calls for the <u>sum</u> of a positive 7 and a negative 10 .

Please return to page $\frac{22}{2}$ and try question 2 again.

$\frac{4}{2}$

From negative 2 we are asked to subtract negative 3 . Then the subtraction rule calls for the addition of the opposite of the subtrahend. That is:

(-2) - (-3)

becomes (-2) + (opposite of -3)

the opposite of -3 is +3

or (-2) + (+3)

Now if you apply the proper addition rule, you will find that this choice is not correct.

Please return to page $\frac{21}{2}$ and try question 10 again.

It appears that you did part of the problem correctly, but then made a mistake. The sum of the positive numbers is +31.9 while the sum of the negative numbers is -57.3 .

Now Rule 4 applies. It tells you to subtract one absolute value from the other. You will probably be more comfortable if we put one number under the other.

$$\begin{array}{r} 57.3 \\ \underline{31.9} \\ 25.4 \end{array}$$

Since this deals with absolute values, we must subtract smaller from larger, and the result is an absolute value.

Please return to page 20/2 and try question 6 again.

The absolute value of

$$p + q$$

is never negative by the definition of absolute value.
But the sum of p and q can be negative when p is positive and q is negative. For example, suppose

$$p = +5$$

and $q = -12$

Then $p + q = -7$

which would not equal the absolute value of 7 . Since there is at least one set of numbers which makes the statement false, this choice is not correct.

Please return to page 23/2 and try question 8 again.

V

$\frac{6}{1}$

Whenever two numbers are written in this manner, it represents an ADDITION.

a	-	b	is rewritten	as	a +	(-b)
7	-	10	becomes		7 +	(-10)
			or		+7 +	(-10)

This problem calls for the addition of positive 7 and negative 10 . Remember that 7 means the same as +7 . The -10 means that the second number is a negative 10 ; you must not think of it as a subtraction example. If you apply the proper addition rule, you will find that this result is not correct.

Please return to page $\frac{14}{2}$ and try this question again.

$\frac{6}{2}$

This problem deals with the sum of positive 7 , negative 12 , and positive 1 . Applying the addition rules , this equals -4 . Performing the addition offered in this choice gives the result -4 . Then this choice is incorrect, since we wanted the one which did NOT equal the original expression.

Please return to page $\frac{13}{2}$ and try question 11 again.

It appears that you multiplied numbers. You must not think that parentheses call for multiplication; they do not.

This problem asks for the sum of positive 2 and positive 7 .

Please return to page $\frac{2}{2}$ and try question 1 again.

The original problem called for the addition of positive x , negative y , and <u>negative z</u> .

Remembering that quantities written in this manner mean an addition, this choice equals the sum of positive x , negative y , and <u>positive z</u> .

Then this choice is not correct.

Please return to page $\frac{28}{2}$ and try question 9 again.

V

This is a perfect illustration of Rule 2 , the addition of two negative numbers. The rule states that the result is to be negative, and that we add the two absolute values.

Remember:

if	$a < 0$ and $b < 0$
then	$a + b = -(\lvert a \rvert + \lvert b \rvert)$

Therefore, this choice is correct.

Please proceed to question 5 below.

Question 5

Apply the proper rule to find the value of

$$18 + 6 - 41$$

(A) 53

(B) -17

(C) 65

(D) None of these.

This problem calls for the addition of a positive 7 and a negative 10 . Then it is necessary to apply Rule 3 and 4 . The sign of the result is the same as the sign of the number with the larger absolute value. Since 10 is the larger absolute value, this result should be negative.

Then this choice is not correct.

Please return to page $\frac{22}{2}$ and try question 2 again.

Let us consider the inequality in two parts. The left side asks for the absolute value of -7 and -8 . By Rule 2 , the sum of -7 and -8 is -15 , and the absolute value of -15 is 15 .

The right side of the expression is equal to 7 - 8 , since we need the absolute values of the quantities.

But, by Rule 4

$$7 - 8 = -1$$

Then this choice says that the left side is smaller than the right side. Upon substituting the values, we find 15 is smaller than -1 , which is certainly not correct.

Please return to page $\frac{29}{2}$ and try question 7 again.

10/1

This was supposed to be an addition of four signed numbers. It appears that you treated it as having a subtraction. Did you subtract 2.6 from 54.7 ? That was incorrect since you were supposed to add -54.7 to -2.6 and to the two positive numbers as well. Remember, you must use addition rules. Let's convert the problem to symbols:

$$a + b - c - d \quad \text{becomes:}$$

$$(a + b) + [(-c) + (-d)]$$

and there are two separate addition problems to be done. Then, these two answers, themselves, form a third addition problem.

Please return to page 20/2 and try question 6 again.

10/2

You were given that p was positive and q was negative. Now the ABSOLUTE VALUE of any number or combination of numbers is never negative by the very definition of ABSOLUTE VALUE. Therefore, $|p + q|$ will be positive, and its value will be bound by subtracting the absolute values of p and q . (Rule 3) On the other hand, the expression $p - q$ means the sum of p and $-q$. The value of this expression will be found by adding the absolute values since,

$p + (-q)$ (when q itself is negative) becomes

$p +$ (the additive inverse of negative-valued q)

NOTE: $-(-5) = 5$

or $p + |q|$ (Rule 1)

Therefore, we see that in the equation $|p + q| = p - q$

(a) the value of one side is found by subtracting the absolute values, and

(B) the value of the other side is found by adding the same absolute values.

The only case where this would produce an equality is when q is zero, and this was ruled out when it was given that q is negative.
Therefore, this choice is not an identity.

Please return to page 23/2 and reconsider this question.

You came very close, but this answer is not correct.

Did you recognize that this is an ADDITION problem? That's right, addition! When two numbers are written in this manner, it should be considered as addition,

$$a - b \quad \text{is rewritten as}$$

$$a + (-b)$$

In this case, we are adding positive 7 and negative 10 . If you will use the proper rule, you will get the correct result. Some students read this problem as " 7 subtract 10 " and since that makes no sense, they consider it to be " subtract 7 from 10 ." Of course, this is completely wrong; the minus sign is the sign of the number, rather than indicating an operation to be performed.

Please return to page 14/2 and try question 3 again.

Since this problem involves addition, it is possible to omit the parentheses without changing the value.

In the following case:

$$a + (b + c) =$$

$$a + b + c$$

This is the reflexive form of the ASSOCIATIVE PROPERTY OF ADDITION. The original expression is equal to this choice. However, this choice is not correct. What further simplification is possible?

Please return to page 26/2 and try question 12 again.

$\frac{12}{1}$

Since this problem asks for the sum of two positive numbers, we apply Rule 1 . The absolute value of the sum is the sum of the absolute values. What did you do with the absolute values?

This choice is not correct.

Please return to page $\frac{2}{2}$ and try question 1 again.

$\frac{12}{2}$

The original problem called for the addition of positive x , negative y , and negative z . But this choice calls for the addition of positive x , positive y and positive z .

Then this choice is not correct.

Please return to page $\frac{28}{2}$ and try question 9 again.

From negative 2 we are asked to subtract negative 3 . Then the subtraction rule calls for the addition of the opposite of the subtrahend.

That is (-2) - (-3)

becomes (-2) + (opposite of -3)

or (-2) + (+3)

Now, we use addition rules 3 and 4 .

(a) The sign of the result is plus.

(b) The absolute value of the sum is the result of subtracting 2 from 3 .

Then the correct result is +1 , and you have chosen the correct answer.

Please proceed to question 11 below.

Question 11

Apply the proper rules to find the choice which is NOT equal to

7 - 12 + 1

(A) 7 - 13

(B) -5 + 1

(C) 8 - 12

(D) 8 - 10 - 2

This example requires the use of Rules 3 and 4 , which tell us that the sign of the result is the same as that of the number which has the larger absolute value. Since 10 is larger than 7 , our result should be negative (the sign of the 10). Also, to get the absolute value of the result, we must subtract absolute values, which gives us 3 .

Therefore, the correct result is -3 , and you have chosen correctly.

Please proceed to question 3 below.

Question 3

Apply the proper rule to find the value of

7 - 10

(A) 17

(B) 3

(C) +3

(D) -3

This problem is an addition of three numbers; the first two are positive, and the third is negative.

The ASSOCIATIVE PROPERTY says that you can group them as you wish, either the first two together or the last two together.

The COMMUTATIVE PROPERTY permits you to change the order of any two numbers. You will end up by having to apply the addition rules twice. The easiest calculation arises from first adding 18 and 6 and then adding the negative 41 .

If you do it that way, you will find that your choice is not correct.

Please return to page $\frac{8}{2}$ and try question 5 again.

Since these quantities are to be added, we may remove the parentheses being careful that the sign of each term is shown.

Thus we have:

$$(x + 2y) + (2x - y) =$$

$$+x + 2y + 2x - y$$

Now if you combine like terms, you will discover that this choice is not correct.

Please return to page $\frac{40}{2}$ and try question 14 again.

V

16/1

Let us consider the inequality in two parts. The left side calls for the absolute value of the sum of -7 and -8 . The sum of -7 and -8 by Rule 2 is -15 , and its absolute value is 15 . The absolute values of -7 and -8 are respectively 7 and 8 , and their sum is 15 . The right side calls for the sum of the individual, absolute values of the same numbers. The left side is said to be larger than the right.

Then this choice states that 15 is larger than 15 which is not correct.

Please return to page 29/2 and try question 7 again.

16/2

Since this problem involves addition, it is possible to omit the parentheses without changing the value.
In the following case:

$$a + (b + c) =$$

$$a + b + c$$

This is the reflexive form of the ASSOCIATIVE PROPERTY OF ADDITION. But this choice indicates that you used the opposite of m - 4 . What caused you to use the opposite?

Note: $a + (b - c) = a + b - c$

but $a + (b - c) \neq a + (c - b)$

However, $a - (b - c)$ does $=$

$a + (c - b)$!

This choice is not correct.

Please return to page 26/2 and try question 12 again.

This problem deals with the sum of two positive numbers. Then we should apply Rule 1 . But this states that the sum is positive, which disagrees with this choice. In addition the rule calls for adding the absolute values, which was not done in this choice. What rule did you apply?

Please return to page 2/2 and try question 1 again.

The original problem calls for the addition of positive x , negative y , and negative z . This choice offers the difference between x and y - z . Let's analyze this to see how it compares with the original problem. Using the subtraction rule, we need the sum of x and the opposite of y - z .

But

$$y - z = y + (-z)$$

and, therefore, the opposite of y - z is -y + z .

That gives us

$$x - y + z$$

as the equivalent of this choice. Then this is the sum of positive x , negative y , and positive z , which does not equal the original expression.

Then this choice is not correct.

Please return to page 28/2 and try question 9 again.

18/1

One of the techniques for making a decision about the truth of an identity is to test it with typical values from the replacement set. If one case is found where the identity doesn't hold, then the statement in question cannot be considered an identity. In order for this statement to be correct, it must be true for any values of p and q . Suppose

$$p = +5 \quad \text{and} \quad q = -12$$

Then, by Rule 4 the sum of p and q is -7 , and its absolute value is 7 .

Now the absolute values of p and q are respectively 5 and 12 , and their sum is 17 . When these values are substituted in the statement in question which says the two parts are always equal, we find a contradiction since 7 does not equal 17 .

This choice is not correct.

Please return to page 23/2 and try question 8 again.

18/2

The original problem is in the form:

$$a - (b - c)$$

This choice is in the form

$$a + (b - c)$$

which is not equivalent.

You have made a mistake in applying the rule for subtraction:

in the case $a - (b - c)$ we change it to an

addition problem $a + [-(b - c)]$

which becomes $a + (-b + c)$ by employing the opposite

and this becomes $a \quad -b + c$

What is the opposite of $(2r - 1)$?

This choice is wrong for another reason. It offers an addition, and we could omit the parentheses and then combine terms. Then this is not the simplest form even if it were a correct expression.

Please return to page 35/2 and try question 13 again.

V

It is important that you recognize a - b to be an example in ADDITION of two signed numbers, (a) + (-b) , positive 7 and negative 10 . Then we can apply Rules 3 and 4 getting the minus sign for the result, since negative 10 has a larger absolute value than positive 7 . The rule tells us to subtract absolute values, which gives us 3 .

Then the correct answer is -3 .

Please proceed to question 4 below.

Question 4

Apply the proper rule to find the value of

(-9) + (-6)

(A) -15

(B) - 3

(C) + 3

(D) +15

This problem is an addition of three numbers, two positive and one negative. You can apply the ASSOCIATIVE and COMMUTATIVE PROPERTIES to make many changes in the appearance of the problem.

However, the easiest calculation results from adding the first two numbers getting 24 , and then adding 24 and -41 .

Applying Rule 2 , we find that our result is -17 , so that this choice is correct.

Please proceed to question 6 below.

Question 6

Perform the calculation to find the value of

$$24.3 + 7.6 - 54.7 - 2.6$$

(A) -25.4

(B) -34.6

(C) -20.2

(D) None of these.

The original problem called for the addition of positive x , negative y , and negative z . Remembering that quantities written in this manner mean an addition, this choice is the sum of positive x , negative y , and negative z .

Then this choice is correct.

Please proceed to question 10 below.

Question 10

Apply the proper rule to find the value of

(-2) - (-3)

(A) +5

(B) -5

(C) +1

(D) -1

V

Since we are looking for the sum of two positive numbers, Rule 1 says that the result is positive; and that the absolute value of the result is the sum of the absolute values.

Then,

$$(+2) + (+7) =$$
$$+ (|2| + |7|) =$$
$$+ 9$$

if	$a > 0 \quad b > 0$
then	$a + b = +(|a| + |b|)$

Please proceed to question 2 below.

Question 2

Apply the proper rule to find the value of

$$(+7) + (-10)$$

(A) - 3

(B) +17

(C) 3

(D) + 3

Let us consider the two parts of the inequality separately. The left side calls for the absolute value of the sum of -7 and 8. The sum of -7 and $+8$ is $+1$ and its absolute value is 1. The right side calls for the sum of the individual absolute values. The absolute values of -7 and 8 are respectively 7 and 8, and their sum is 15.

Then this choice states that 1 is smaller than 15, which is correct.

Please proceed to question 8 below.

Question 8

If $p \in \{\text{positive integers}\}$ and $q \in \{\text{negative integers}\}$ which choice do you recognize as an identity?

(A) $|p + q| = p + q$

(B) $|p + q| = p - q$

(C) $|p + q| = |p| + |q|$

(D) None of these.

$\frac{24}{1}$

You have done a good job of applying the rule for subtraction since you are adding the opposite of the subtrahend. However, in the form

$$a + (-b + c)$$

the parentheses may be omittted getting

$$a - b + c$$

which would result in having terms that could be combined. Then this choice is not correct, because it is not in simplest form.

Please return to page $\frac{35}{2}$ and try question 13 again.

$\frac{24}{2}$

This problem requires that the like terms be combined by using the appropriate addition rules. It might be advisable to rearrange the terms first using the COMMUTATIVE PROPERTY so as to group the like terms.

Thus:

$$2m - 3 + 5m - 10m + 8 =$$

$$2m + 5m - 10m - 3 + 8$$

If you now combine terms, you will discover that this choice is not correct.

Please return to page $\frac{31}{2}$ and try question 15 again.

If you follow carefully the rules for addition of signed numbers, you will find that one of the other choices is correct. This choice is not correct.

Consider the problem symbolically:

$$a + b - c - d$$

becomes $(a + b) + [(-c) + (-d)]$

this becomes

$$e + f$$

Thus we are faced with three separate addition problems.

Please return to page $\frac{20}{2}$ and try question 6 again.

$$\boxed{a - b = a + (\text{additive inverse of } b)}$$

Applying the subtraction rule, we have:

$$(2w - 5x) - (-3w + 2x) =$$
$$(2w - 5x) + (+3w - 2x)$$

Now, we can remove parentheses and combine terms; and we find that this choice is not the correct result.

Please return to page $\frac{32}{2}$ and try question 17 again.

26
1

The problem deals with the sum of positive 7 , negative 12 , and positive 1 . We can check each solution offered as a choice by applying the addition rules. The original problem equals -4 . Applying the addition rules to this choice, we do not get -4 . As a matter of fact, we get -6 . Since we wanted the choice which was NOT equal to the original, this choice is correct.

Remember, we can use the COMMUTATIVE LAW FOR ADDITION, and then group two adjacent terms by the ASSOCIATIVE LAW. This gives us much latitude for different approaches to the solution.

Note:

$$7 - 12 + 1 = (7 + 1) - 12 \quad \text{or}$$

$$= (7 - 12) + 1 \quad \text{or}$$

$$= 7 + (-12 + 1)$$

but $7 - 12 + 1 \neq 7 - (12 + 1)$!

Please proceed to question 12 below.

26
2

Question 12

Apply the proper rules to simplify the expression

$$32 + (m - 4)$$

as far as possible.

(A) $32 + m - 4$

(B) $m + 28$

(C) $36 - m$

(D) $28m$

You have done part of the problem correctly. However, you overlooked something. The rule for subtraction tells us to add the opposite of the subtrahend. You seem to have made a mistake in getting the opposite of

$$2r - 1$$

What is the opposite of -1 ?

Please return to page 35/2 and try question 13 again.

There are two ways to solve this type problem.

(1) Simplify first, then substitute and further simplify.

or (2) Substitute first, then simplify.

Using the second method and substituting the given values, we have:

$$(p + q) - p =$$

$$(6 - 18) - 6$$

If you now calculate the value of the expression in parentheses and then add the negative 6 , you will find that this result is not correct.

Please return to page 37/2 and try question 16 again.

V

A careful check of each of the other choices shows that none of them is correct for all values of p and q. Then this choice is correct.

Please proceed to question 9 below.

Question 9

Apply the proper rules to find the choice which is equal to

$$(x) + (-y) + (-z)$$

if x, y, and z are all positive integers.

(A) $x - y + z$

(B) $x + y + z$

(C) $x - y - z$

(D) $x - (y - z)$

You have done a fine job; this choice is correct.

While this can be done in many ways, it is best if you first add the two positive quantities, then the two negative quantities. For convenience, we may write it as follows:

	24.3	- 54.7	One case uses Rule 1
+	7.6	- 2.6	and the other uses
	31.9	- 57.3	Rule 2 .

Now we need to subtract absolute values, and you are probably most comfortable with the larger absolute value on top.

57.3	Applying Rules 3 and 4 , we find that
31.9	our result is negative so that we
25.4	get -25.4 .

Please proceed to question 7 below.

Question 7

Perform the calculation to find which inequality is correct.

(A) $|(7) + (-8)| > |7| + |-8|$

(B) $|-7 \quad -8| < |-7| - |-8|$

(C) $|(-7) + (-8)| > |-7| + |-8|$

(D) $|-7 + 8| < |-7| + |8|$

$\frac{30}{1}$

Since this problem involves addition, it is possible to omit the parentheses without changing the value.

In the case:

$$a + (b + c) =$$

$$a + b + c$$

This is the reflexive form of the ASSOCIATIVE PROPERTY OF ADDITION.

Then,

$$32 + (m - 4) =$$

$$32 + m - 4$$

and combining terms

we get $28 + m$

How did you combine these terms? Or did you select this choice because you didn't notice that it does not have the plus sign? In any case, this choice is not correct.

Please return to page $\frac{26}{2}$ and try question 12 again.

$\frac{30}{2}$

The rule for subtraction tells us to add on the opposite of the subtrahend. What is the opposite of

$$(3r - s + 9) \quad ?$$

You have made an error in signs.

Please return to page $\frac{39}{2}$ and try question 18 again.

Since these quantities have to be added, we may remove the parentheses being careful to show the sign of each term.

Thus we have:

$$(x + 2y) + (2x - y) =$$

$$+x + 2y + 2x - y =$$

$$+ 3x + y$$

Of course, the plus sign in front of the $3x$ is written for emphasis; it may be omitted without changing the value.

Therefore, this choice is correct.

Please proceed to question 15 below.

Question 15

Apply your knowledge to combine terms:

$$2m - 3 + 5m - 10m + 8$$

(A) $3m + 5$

(B) $-3m + 5$

(C) $3m - 11$

(D) $-3m - 11$

V

We can solve this problem by substituting the given values.

Thus,

$$(p + q) - p = (6 - 18) - 6 \qquad \text{Rule } 4$$
$$= -12 - 6 \qquad \text{Rule } 2$$
$$= -18$$

It turns out that the problem is easier if we do some algebra first:

$$(p + q) - p = p + q - p \quad [\text{COMMUTATIVE PROPERTY}$$
$$= p - p + q \quad [\text{ADDITIVE INVERSE}$$
$$= 0 + q$$
$$= q$$

Notice that in using the variables p and q , we do not care whether they represent positive or negative numbers. Of course, the result is the same whichever approach we use since all correct methods give us the correct answer.

Please proceed to question 17 below.

Question 17

Apply your knowledge to combine terms:

$$(2w - 5x) - (-3w + 2x)$$

(A) $5w - 3x$

(B) $5w + 7x$

(C) $5w - 7x$

(D) $-w - 3x$

Since I is an addition, we have

$$x + (y - z) = x + y - z$$

Item II is a subtraction, therefore,

$$x - (y + z) = x(-y - z)$$

which equals $x - y - z$

Therefore, I and II are not equal, and this choice is not correct.

Please return to page 45/2 and try question 19 again.

You have answered a different question correctly, but you were asked for the absolute value of the product of the two numbers. An absolute value does not have a sign attached.

What is the rule for finding the absolute value of a product?

Please return to page 47/2 and try question 2 again.

34/1

Since this choice has two quantities separated by a minus sign, it indicates an addition of the signed numbers, a and -1 . But the original problem did not have a plus or a minus sign separating the two quantities; it was a multiplication problem in the form

b (d) which means b x d

The fact that d was a negative value doesn't change the operation to subtraction or addition of a negative. It is still multiplication.

Since the addition and the multiplication do not give the same values, this choice is not correct.

Please return to page 42/2 and try question 1 again.

34/2

The numbers, each in parentheses, are written without a plus or a minus sign between them; this indicates a multiplication.

NOTE:	$(-a)(-b) = +(\lvert a\rvert \cdot \lvert b\rvert)$

You have made an error in one part of the rule for multiplying two numbers. Do you see which part?

Please return to page 58/2 and try the question again.

Since this problem involves addition, it is possible to omit the parentheses without changing the value.

In the following case:

$$a + (b + c) = a + b + c$$

This is the ASSOCIATIVE PROPERTY OF ADDITION in its reflexive form.

Then we have:

$$\begin{aligned} 32 + (m - 4) &= 32 + m - 4 \\ &= 32 - 4 + m \quad \text{Using the COMMUTATIVE PROPERTY} \\ &= 28 + m \quad \text{Combining similar terms} \\ &= m + 28 \quad \text{Using the COMMUTATIVE PROPERTY} \end{aligned}$$

Then this choice is correct since the two terms we now have cannot be combined.

Please proceed to question 13 below.

Question 13

If you apply the proper rules to the problem

$$5r - (2r - 1)$$

which is the result in simplest form?

(A) $5r + (2r - 1)$

(B) $5r + (-2r + 1)$

(C) $3r - 1$

(D) $3r + 1$

Since P represents the sum of three numbers which might be either negative or positive or zero, P may be positive or negative (or even zero). But Q is an absolute value, and it cannot be negative. Therefore, this choice is not correct. If you would be happier trying some numbers, suppose that

$$m = 1$$

$$t = 2$$

and $$w = 6$$

The value of P would be

$$1 + 2 - 6 = -3$$

The value of Q would be the absolute value of -3, which is 3.

Please return to page 38/2 and try question 20 again.

Since the quantities are not separated by a sign, either plus or minus, this indicates a multiplication. The rule for multiplication has two parts; one for the sign of the result, and the other for the absolute value of the result.

NOTE:

$$\boxed{a\,(-b) = -(|a| \cdot |b|)}$$

Which did you use incorrectly?

This result is not correct.

Please return to page 52/2 and try question 3 again.

This problem requires that the like terms be combined using the appropriate addition rules. It is best to rearrange terms first using the COMMUTATIVE PROPERTY so as to group the like terms.

Thus:

$$2m - 3 + 5m - 10m + 8 = \underbrace{2m + 5m} - 10m - \underbrace{3 + 8}$$

$$= \underbrace{7m - 10m} + 5$$

$$= -3m + 5$$

Therefore, this choice is correct.

Please proceed to question 16 below.

Question 16

If

$$p = 6$$

$$q = -18$$

apply your knowledge to find the value of

$$(p + q) - p$$

(A) -20

(B) 18

(C) - 6

(D) -18

V

38/1

Since I is an addition, we have

$$x + (y - z) = x + y - z$$

Item III , which cannot be simplified is

$$x - y - z$$

Then these are not equal, and this choice is not correct.

Please return to page 45/2 and try question 19 again.

38/2

The parentheses indicate that the two quantities 4 and -3 are not separated by any signs, and that they are to be multiplied.

NOTE:

(a) (b) means a x b

But you did not multiply them; therefore, this choice is not correct.

Please return to page 47/2 and try question 2 again.

$$a - b = a + (\text{Additive Inverse of } b)$$

NOTE: Additive Inverse of: $-p + q$ is $+p - q$

Applying the subtraction rule, we have:

$$(2w - 5x) - (-3w + 2x) = (2w - 5x) + (+3w - 2x)$$
$$= +2w - 5x + 3w - 2x \quad [\text{ Combine like terms}$$
$$= 5w - 7x$$

Then this choice is correct. Notice that it is possible to combine the similar terms without first rewriting the expression to put the similar terms next to each other. However, you must take care not to overlook any term.

Please proceed to question 18 below.

Question 18

In performing the calculation

$$7r - (3r - s + 9)$$

choose the expression which must be added to $7r$.

(A) $-3r + s - 9$

(B) $-3r - s + 9$

(C) $-3r - s - 9$

(D) $3r - s + 9$

40
1

The rule for subtraction tells us to add the opposite of the subtrahend. Then,

$$5r - (2r - 1)$$

becomes: $$5r + (-2r + 1)$$

and we can omit parentheses from this addition without changing the value. This gives us

$$5r - 2r + 1$$

Combining terms, we have

$$3r + 1$$

It is very important that you see what happened to the plus sign which was in front of the parentheses.
The meaning of the plus sign was "add" and the meaning of writing

$$5r - 2r + 1$$

is also "add". Therefore, when the parentheses are omitted as they may be in addition, the plus sign is also omitted. However, we must be sure to write the sign of the first term in the parentheses. For instance,

$$7x + (2x + 1) = 7x + 2x + 1$$

where the plus sign of the $2x$ was understood inside the parentheses, but must be written when the parentheses are omitted. In the problem we started with, the minus sign in front of the $2r$ is written at all times.

Please go on to question 14 below.

40
1

Question 14

Apply the proper rule to express in simplest form:

$$(x + 2y) + (2x - y)$$

(A) $3x + 3y$

(B) $3x - y$

(C) $3x + y$

(D) $-x + 3y$

Neither Q nor R can be negative quantities since one is an absolute value and the other is the sum of two absolute values. Nevertheless, we can show that they do not have to be equal.

Suppose

$$m = 1$$
$$t = 2$$
and $$w = 6$$

Then,

$$Q = |1 + 2 - 6| \qquad R = |1| + |2 - 6|$$
$$= |-3| \qquad = 1 + |-4|$$
$$= 3 \qquad = 1 + 4$$
$$= 5$$

Therefore, this choice is not correct.

Please return to page 49/2 and reconsider the problem.

Since the quantities are not separated by either a plus or minus sign, this indicates a multiplication. The rule for multiplying two signed numbers has two parts; one for the sign of the result, and the other for the absolute value of the result.

NOTE:

$$a(-b) = -(|a| \cdot |b|)$$

You have made a mistake in one of those parts; this result is not correct.

Please return to page 52/2 and try question 3 again.

V

VOLUME 5 SEGMENT 2 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2	2 (Sequence Number)
	54	and	56	0	4 (Type of Punch Card)
	60	and	62	0	5 (Volume Number)
	66	and	68	0	2 (Segment Number)

Your READING ASSIGNMENT for this Segment is pages: 133 - 135.

SUPPLEMENTARY NOTES:

In the previous segment you saw how to add and subtract signed numbers without necessarily referring to the NUMBER LINE. You will now extend this skill to include multiplication of signed numbers. This is a logical development since multiplication can be considered as repeated addition.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which do you recognize as the same value as

$$a\,(-1)$$

(A) $a - 1$

(B) $(a) - 1$

(C) $-1a$

(D) $-1 + a$

V

You have correctly applied the COMMUTATIVE LAW OF MULTIPLICATION, but you still have to actually multiply the numbers in order to simplify the expression.

Please return to page 58/2 and finish the problem.

You must not concentrate so hard on one part of the problem that you overlook something else. This problem calls for the multiplication of four numbers. However, you can only multiply two numbers at a time. The ASSOCIATIVE PROPERTY tells you that you can multiply two numbers, then multiply the result by the next number and so on.

If you do that correctly, you will find that this choice is not correct.

Please return to page 65/2 and try question 7 again.

V

$\frac{44}{1}$

Since P is the sum of three signed numbers, it may have any value, positive, negative or zero. On the other hand, since Q is an absolute value, it must be non-negative. Also R is the sum of two absolute values and cannot be negative.

Then it is certainly not true that all three are equal.

Please return to page $\frac{49}{2}$ and try question 20 again.

$\frac{44}{2}$

Since no plus or minus sign separates the two quantities, this is a multiplication. What operation did you perform?

This choice is not correct.

Please return to page $\frac{63}{2}$ and try question 5 again.

The rule for subtraction tells us to add on the opposite of the subtrahend, that is, the quantity being subtracted. In this case the subtrahend is

$$(3r - s + 9)$$

Then its opposite is

$$-3r + s - 9$$

and this choice is correct.

Please proceed to question 19 below.

Question 19

Apply your knowledge to find which statement about the following expressions is correct.

I. $x + (y - z)$

II. $x - (y + z)$

III. $x - y - z$

(A) only I and III are equal

(B) only II and III are equal

(C) only I and III are equal

(D) all three are equal

46
1

Since the quantities are not separated by either a plus or a minus sign, this indicates multiplication. The rule for multiplying two signed numbers has two parts, one for the sign and the other for the absolute value of the result.

NOTE:

$$a(-b) = -(|a| \cdot |b|)$$

You have an error in both parts; this choice is not correct.

Please return to page 52/2 and try question 3 again.

46
2

You seem to be confused.

$$-3\ -5\ -2$$

calls for the addition of the negative numbers; where as

$$(-3)\ (-5)\ (-2)$$

indicates multiplication. Now examine the rule for multiplication.

a b c means a x b x c

By the COMMUTATIVE LAW, the order of these can be changed:

a x c x b

c x b x a etc.

without affecting the result. Therefore, multiply any two, and then multiply that answer by the third. Consider the signs carefully in each case.

Please return to page 56/2 and try question 6 again.

This choice indicates a multiplication since the two quantities are written without a plus or minus sign between them. It is written in the form b (d) which means b x d . This is true whether or not either b or d are actually negative numbers. This is, therefore, another form of the given expression.

Since the COMMUTATIVE PROPERTY states that the result of multiplication is the same regardless of the order of the quantities, these have the same value.

Then this choice is correct.

Please proceed to question 2 below.

Question 2

Choose the absolute value of

(4) (-3)

(A) -12

(B) 12

(C) 1

(D) - 1

$\frac{48}{1}$

The fact that the two numbers each in parentheses are written without a plus or a minus sign separating them indicates that this is a multiplication.

What operation did you perform? You certainly did not perform a correct multiplication.

Please return to page $\frac{58}{2}$ and try the question again.

$\frac{48}{2}$

How did you read the problem? It asks for the value of

$$(+2)^3 \quad \text{not} \quad (+2)3$$

Remember that the base is to be repeated as a factor in a multiplication. That is, the base +2 is to be written three times and the three numbers must then be <u>multiplied</u>.

$$(a)^3 \quad \text{means} \quad (a)\ (a)\ (a)$$

Your choice is not correct.

Please return to page $\frac{53}{2}$ and try question 8 again.

Since I is an addition, we have

$$x + (y - z) = x + y - z$$

Item II is a subtraction, therefore,

$$x - (y + z) = x + (-y - z)$$

Which equals

$$x - y - z$$

Item III , which cannot be simplified; is

$$x - y - z$$

Therefore, II and III are equal, but I is different.

Then this choice is correct.

Please proceed to question 20 below.

Question 20

Apply your knowledge to find which statement about the following expressions is correct: m , t , and w are integers,

$P = m + t - w$ $\quad Q = |m + t - w|$ $\quad R = |m| + |t - w|$

(A) only P and Q are equal

(B) only Q and R are equal

(C) all three are equal

(D) all three may have different values

$\frac{50}{1}$

It appears that you recognized correctly that this is a multiplication problem. Why did you get the wrong answer? Perhaps you should reread the rules for multiplication; you must have left out one part of the rule.

Please return to page $\frac{56}{2}$ and try question 6 again.

$\frac{50}{2}$

In this problem the base is -2 and the exponent is 4 .

Then writing the problem as a multiplication gives us

(-2) (-2) (-2) (-2)

If you apply the multiplication rules for signed numbers, you will find that this choice is not correct. You may use the ASSOCIATIVE LAW and group the factors in pairs. Then each pair can be replaced by the equivalent product. These two products are then multiplied.

Please return to page $\frac{67}{2}$ and try question 9 again.

It does not follow that since multiplying two negative numbers produces a positive number, multiplying two positive numbers produces a negative number.

Refer to the proper rule and then apply it.

Please return to page 63/2 and try question 5 again.

Since this problem involves a multiplication of a single term by an expression made up of two terms, you must apply the DISTRIBUTIVE PROPERTY; that is, you must use the multiplier with each term inside the parentheses. You certainly have not done that.

$$a (b + c) = ab + ac$$

Please return to page 69/2 and try question 11 again.

V

52
1

Since the rule states that the absolute value of a product is the product of the absolute values, it is only necessary to multiply 4 by 3 .

Note: $\boxed{|(a)(b)| = |a| \times |b|}$

Then this choice is correct.

Please proceed to question 3 below.

52
2

Question 3

Choose the value of

$$4(-3)$$

(A) -12

(B) 12

(C) 1

(D) - 1

V

There is a lot of detail in this problem, and you have handled it all carefully and correctly. Once you know that it is a multiplication, you can proceed as follows:

> There are four negative numbers, an even number of them, so that the result is plus. (Each multiplication of two negatives produces a positive product). Now you can multiply absolute values without having signs in your way. Of course, you may do the multiplication in any order you wish.

Please proceed to question 8 below.

Question 8

Apply your knowledge to find the value of $(+2)^3$

(A) 6

(B) 8

(C) 9

(D) 12

The fact that each quantity differs from the others in the placement of absolute value signs suggests that they have different values.

Let $m = 1$

$t = 2$

$w = 6$

Then

$P = m + t - w$	$Q = m + t - w$	$R = \lvert m \rvert + \lvert t - w \rvert$
$P = 1 + 2 - 6$	$Q = \lvert 1 + 2 - 6 \rvert$	$R = \lvert 1 \rvert + \lvert 2 - 6 \rvert$
$P = -3$	$Q = \lvert -3 \rvert$	$R = 1 + \lvert -4 \rvert$
	$Q = 3$	$R = 1 + 4$
		$R = 5$

Thus, we find that all three can have different values.

You have now finished this Segment. Hand in the PUNCH CARD.
You should have entered in your NOTEBOOK the rules for addition and subtraction from the beginning of this segment.

You should now be able to complete assignment 5 , problems 1 - 4 .

In this problem the base is -2 and the exponent is 4 .

Therefore, we are asked for the value of

$$(-2)(-2)(-2)(-2)$$

There are four negative factors; therefore, the result is positive.

The product of the absolute values is 16 so that this choice is correct.

You might note this suggestion for the calculation:

$$\underbrace{(2)(2)}_{4}\underbrace{(2)(2)}_{4} = 4 \cdot 4 = 16$$

Please proceed to question 10 below.

Question 10

Apply your knowledge to find the value of

$$(-2)^5$$

(A) 32 (C) 10

(B) -32 (D) -10

$\frac{56}{1}$

Since the expression is in the form:

$$+a\ (\ +b\ )$$

this is a multiplication.

You have applied the rule correctly, and this is the correct choice.

$$+a\ (\ +b\ ) \quad = \quad +a \quad \times \quad +b$$

Please proceed to question 6 below.

$\frac{56}{2}$

Question 6

Apply your knowledge to find the value of

$$(\ -3\ )(\ -5\ )(\ -2\ )$$

(A) -10 (C) -30

(B) 10 (D) 30

57/1

This problem requires the application of the DISTRIBUTIVE PROPERTY as you have recognized. But you have made an error; this choice is not correct. Did you forget to apply the rules for multiplying <u>signed</u> numbers?

Please return to page 69/2 and try question 11 again.

57/2

Applying the DISTRIBUTIVE PROPERTY correctly, we get:

$$5(3a) + 5(r) - 4(3a) - 4(r)$$

If you perform the multiplications correctly and then combine terms, you will find that this choice is not correct.

Please return to page 84/2 and try question 14 again.

V

Since the quantities are not separated by a sign, either plus or minus, this indicates a multiplication. Then the rule for multiplication tells us that the result is negative, and its absolute value is 12 .

Note:

$$a(-b) = -(|a| \cdot |b|)$$

Therefore, this choice is correct.

Please proceed to question 4 below.

Question 4

Choose the value of

$$(-5)(-1)$$

(A) - 5

(B) 5

(C) $(-1)(-5)$

(D) - 6

Once you recognize that there is an exponent, you should rewrite the problem to show the multiplication. When you try to multiply quantities in your mind without having them on your paper, it is too easy to make errors as you have done.

Since the base is +2 and the exponent is 3 the problem is to find the value of

$$(+2)(+2)(+2)$$

Please return to page 53/2 and try question 8 again.

This problem calls for the DISTRIBUTIVE PROPERTY.

Applying the property correctly means that each term in the parentheses is to be multiplied by the $-3x$.

Therefore, this choice is not correct.

Please return to page 73/2 and try question 13 again.

$\frac{60}{1}$

The base is -2 and the exponent is 5 so that this problem is equivalent to

$$(-2)(-2)(-2)(-2)(-2)$$

If you now perform the multiplication using the rules correctly, you will find that this choice is not correct.

Please return to page $\frac{55}{2}$ and try question 10 again.

$\frac{60}{2}$

The meaning of

$$5x^3 \text{ is } 5 \cdot x \cdot x \cdot x$$

Therefore we have

$$5x^3 = 5(-3)(-3)(-3)$$

Did you forget the meaning of an exponent? If you are careful to write the multiplication out as shown above, you will get the correct result more often.

This result is not correct.

Please return to page $\frac{81}{2}$ and try question 16 again.

This problem calls for the DISTRIBUTIVE PROPERTY first, giving us

$$3m(2m) + 3m(-5) - 4(3) - 4(-2m)$$

Next you should perform the multiplications. This seems to be the place where you made the error. Have you found it?

Please return to page 71/2 and try question 15 again.

A common error in this problem is to consider that it asks for the third power of a negative quantity, and that the result is, therefore, negative. You must remember that a minus sign merely indicates the opposite of a number. Of course -3 is a negative number, but $-m$ is a positive number if m happens to be negative.

Go back to the meaning of a third power, and calculate the value correctly since you have the wrong choice.

Please return to page 79/2 and try question 18 again.

62
1

The problem requires the use of the DISTRIBUTIVE PROPERTY as you have recognized.

$$a(b + c) = ab + ac$$

It is important to keep in mind also that we are dealing with signed numbers, and to follow the rules for multiplying them. You have done very well; this choice is correct.

In detail, we might show the work as follows:

$$-5(2x - 3y) =$$
$$-5(2x) -5(-3y) =$$
$$-10x \quad +15y$$

Please proceed to question 12 below.

62
2

Question 12

Apply the appropriate principle to find the choice equal to

$$-5 - (2x - 3y)$$

(A) $-10x + 15y$

(B) $10x + 15y$

(C) $10x - 3y$

(D) None of these.

The numbers each in parentheses are written without a plus or minus sign between them; this indicates a multiplication. The rule for multiplication has two parts: The absolute value of the product is the product of the absolute values, and the result is negative if the two numbers have different signs.

Note:

$$(-a)(-b) = +(|a| \cdot |b|)$$

Then you have used both parts of the rule correctly, and this choice is correct.

Please proceed to question 5 below.

Question 5

Choose the value of

$$+4(+3)$$

(A) 12

(B) + 7

(C) 7

(D) -12

$\frac{64}{1}$

It appears that you used the DISTRIBUTIVE PROPERTY correctly, and you have no mistakes in multiplying the signed numbers.

Then where did you make your mistake?

Please keep in mind: the more complicated an example becomes, the more careful you must be with all details.

Can you find your error?

Please return to page $\frac{73}{2}$ and try question 13 again.

$\frac{64}{2}$

You have made a mistake probably due to doing too much mentally.

You should write down the multiplication that is to be performed,

$$-2p^2 \text{ means } -2(\,p\,)(\,p\,)$$

Then you can substitute the numbers and perform the multiplication.

This choice is not correct.

Please return to page $\frac{75}{2}$ and try question 17 again.

You have done a fine job!

First, because you recognized that this is a multiplication example. Next, because you applied the rule properly and carefully. It is interesting to note that you can make use of the ASSOCIATIVE AND COMMUTATIVE PROPERTIES OF MULTIPLICATION to simplify this calculation. For example, it can be done as:

(a)	(b)	(c)
$(-3)(-5)(-2) =$	$(-3)(-5)(-2) =$	$(-3)(-2)(-5) =$
$-3 \cdot -5 \cdot -2 =$	$-3 \cdot -5 \cdot -2 =$	$-3 \cdot -2 \cdot -5 =$
$-3 \cdot +10 =$	$+15 \cdot -2 =$	$+6 \cdot -5 =$
-30	-30	-30

Of course, it could be done in other orders as well, all giving the same correct answer.

Please proceed to question 7 below.

Question 7

Apply the appropriate rule to find the value of

$$(-3)(-2)(-1)(-2)$$

(A) -8 (C) -12

(B) 8 (D) 12

$\frac{66}{1}$

This problem calls for the DISTRIBUTIVE PROPERTY first, giving us:

$$3m\ (\ 2m\ )\ +\ 3m\ (\ -5\ )\ -\ 4\ (\ 3\ )\ -\ 4\ (\ -2m\ )$$

Next, you should perform the multiplications and then combine similar terms.

It appears that you have done all that, and yet this choice is not correct. Where did you make your error? Better review each step carefully.

Return to page $\frac{71}{2}$ and try question 15 again.

$\frac{66}{2}$

You have apparently ignored the parentheses in the original problem. It is

$$(\ -2p\ )^3 \quad \text{not} \quad -2p^3$$

If you will re-examine the meaning of a third power, you should be able to correct your error.

Return to page $\frac{79}{2}$ and try question 18 again.

Since the exponent 3 tells us that the base is to be repeated three times as a factor, we have

$$(+2)(+2)(+2) = +8$$

This is an example of

$$\boxed{(a)^3 = (a)(a)(a)}$$

Then this choice is correct.

Please proceed to question 9 below.

Question 9

Apply the proper principles to find the value of $(-2)^4$.

(A) 16

(B) -16

(C) 8

(D) - 8

$\frac{68}{1}$

Since there is a minus sign separating the -5 from the quantity inside the parentheses, the -5 is <u>not</u> to be multiplied by the other quantity.

Then this choice is not correct.

When two quantities are separated by a plus or minus sign, it calls for application of the addition rules.

<u>Note</u>:

$$a - (b - c) \neq a(b - c)$$

Return to page $\frac{62}{2}$ and try question 12 again.

$\frac{68}{2}$

The meaning of

$$5x^3 \quad \text{is} \quad 5(x)(x)(x)$$

Then if you substitute -3 for x it is possible to calculate the value.

However, you will find that this choice is not correct.

Return to page $\frac{81}{2}$ and try question 16 again.

Since the base is -2 and the exponent is 5 , this problem is equivalent to

$$(-2)(-2)(-2)(-2)(-2) =$$

$$(+4) \quad (+4) (-2) =$$

$$(+4) \quad (-8) =$$

-32

There is no reason why you must do the multiplication in the order shown above. In any order the multiplication will come out to this same value since it is correct. You could save some work by seeing that there are five negative numbers (an odd number) , which means that the result is negative. Then you can concentrate on the absolute values without any signs to bother you.

Please proceed to question 11 below.

Question 11

Apply your knowledge to find the choice equal to

$$-5 (2x - 3y)$$

(A) $-10x - 3y$

(B) $-10x + 3y$

(C) $10x - 15y$

(D) $-10x + 15y$

$\frac{70}{1}$

There is a difference between $-2p^2$ and $(-2p)^2$.

Which was the one you used?

Your choice is not correct.

Please return to page $\frac{75}{2}$ and try question 17 again.

$\frac{70}{2}$

Were you tricked by the $-p$?

Since the value of p is -6, what is the value of $-p$?

That may explain why your choice is not correct.

Please return to page $\frac{85}{2}$ and try question 19 again.

Applying the DISTRIBUTIVE PROPERTY correctly, we get:

$$5(3a) + 5(r) - 4(3a) - 4(r) =$$

$$15a + 5r - 12a - 4r =$$

$$3a + r$$

Therefore, this choice is correct.

Please proceed to question 15 below.

Question 15

Apply the proper principles to express as simply as possible:

$$3m(2m - 5) - 4(3 - 2m)$$

(A) $6m^2 - 15m - 12 - 8m$

(B) $6m^2 - 23m - 12$

(C) $6m^2 - 15m - 12 + 8m$

(D) $6m^2 - 7m - 12$

$\frac{72}{1}$

You must learn to distinguish between

$$5x^3 \quad \text{and} \quad (5x)^3$$

The first means

$$5(x)(x)(x)$$

while the second means $(5x)(5x)(5x)$

You have apparently used the second meaning, which is not what the problem called for.

Please return to page $\frac{81}{2}$ and try question 16 again.

$\frac{72}{2}$

There is an error somewhere in your calculation, since one of the other choices is correct.

Are you sure of the proper theory?

$$(-ky)^3 \quad \text{means} \quad (-ky)(-ky)(-ky)$$

Now when y is itself negative, do you see how it affects the sign of each expression in the parentheses?

Please return to page $\frac{79}{2}$ and try question 18 again.

The minus sign separating the -5 from the quantity $2x - 3y$ indicates that this can be considered an addition of a negative number: adding -5 and the opposite of $2x - 3y$. Then we have

$$-5 - (2x - 3y) =$$
$$-5 - 2x + 3y$$

Since this is not equal to any of the other choices, this choice is correct.

We might also interpret the problem as:

$$-5 - (2x - 3y) =$$
$$-5 - 1 (2x - 3y) =$$
$$-5 - 2x + 3y$$

Notice that we may assume the coefficient in front of the parentheses to be 1 since it was not written. Then multiplying by -1 gives us the same result as before.

$- (a - b)$ by additive inverse $= -a + b$

$-1 (a - b)$ by the DISTRIBUTIVE LAW $= -a + b$

Isn't is amazing that we get the same result using different methods? Not really. Any correct method must give the correct result, and there is only one.

Please proceed to question 13 below

Question 13

Apply the proper rule to find the choice equal to

$$(-3x)(-5x + 2y - z)$$

(A) $15x^2 + 2xy - z$

(B) $15x - 6xy + 3xz$

(C) $15x^2 - 6xy + 3xz$

(D) None of these.

V

$\frac{74}{1}$

This problem requires the combination of multiplication and addition, and it is therefore necessary to do the multiplication first. Let us consider each term as a product of factors. We have:

$$3x^2 + 2xy - y^2 =$$

$$3(x)(x) + 2(x)(y) - (y)(y)$$

Now it would be proper to substitute the given values in this expression and do all the multiplications, being careful not to omit or make an error in signs. Then you would need to combine the values of the terms by means of the addition rules. If you do this correctly, this is not the choice you arrive at.

Please return to page $\frac{80}{2}$ and try question 20 again.

$\frac{74}{2}$

If you will review the procedure for solving an equation, you will find that subtraction is used to remove a quantity which has been added to the variable. How is the m attached to the variable? Then what operation of mathematics can be used to remove it? You should choose the inverse operation.

Please return to page $\frac{83}{2}$ and try question 1 again.

The meaning of

$$5x^3 \quad \text{is} \quad 5 (x)(x)(x)$$

Therefore, we have

$$\begin{aligned} 5x^3 &= 5 (-3)(-3)(-3) \\ &= (- 15) \quad (+ 9) \\ &= - 135 \end{aligned}$$

Then this choice is correct.

Please proceed to question 17 below.

Question 17

Apply your knowledge to find the value of

$$-2p^2 \quad \text{when} \quad p = 5$$

(A) -50 (C) -100

(B) 50 (D) 100

$\frac{76}{1}$

If you concentrate very hard to avoid making an error in one part of the problem, you must not relax your caution in the other parts of the problem. Calculate the value of m^2 , then the value of $3m$, and then the value of $-p$.

Now it is necessary to put the parts together. Somewhere along the way you have made a mistake.

Return to page $\frac{85}{2}$ and try question 19 again.

$\frac{76}{2}$

If we attempt to check this equation using the listed value, we get:

$$a \cdot \frac{a}{b} = \frac{a^2}{b}$$

But the equation you chose said that

$$a \cdot x = b$$

Therefore, this choice is incorrect.

Return to page $\frac{98}{2}$ and try question 2 again.

This problem calls for the DISTRIBUTIVE PROPERTY first, giving us:

$$3m\ (\ 2m\ )\ +\ 3m\ (\ -5\ )\ -\ 4\ (\ 3\ )\ -\ 4\ (\ -2m\ )$$

Next you should perform the multiplications. It appears that you have done that. Then why is this choice not correct?

Have you simplified as far as possible?

Return to page 71/2 and try question 15 again.

Your answer does equal the given expression, but your answer to the question is wrong. The question asked for the absolute value of the quotient.

You must be more alert when reading questions.

Return to page 106/2 and try question 4 again.

V

$\frac{78}{1}$

You should recognize that this problem requires the application of the DISTRIBUTIVE PROPERTY as well as the rules for signed numbers. If you work with these very carefully, you will find that one of the other choices is correct.

Return to page $\frac{73}{2}$ and try question 13 again.

$\frac{78}{2}$

This problem calls for the multiplication of $5x$ by the reciprocal of $3s$. Then, according to the definition of division, it is equivalent to the quotient of these quantities.

$$a \cdot \frac{1}{b} = a \div b$$

Then this choice is not correct.

You might recall that $5x$ can be written as a fraction $\frac{5x}{1}$.

Then we would have: $\frac{5x}{1} \cdot \frac{1}{3s} = \frac{(5x)(1)}{(1)(3s)}$

$$= \frac{5x}{3s}$$

We still find that this choice is not correct.

Please return to page $\frac{91}{2}$ and try question 3 again.

V

Since $-2p^2$ means $-2 (p)(p)$, we note that the exponent applies only to the base, p . Substitution of 5 for p makes it equal to

$$-2 (5)(5)$$

Then this choice is correct.

By using the ASSOCIATIVE PROPERTY OF MULTIPLICATION:

$-2 (5)(5)$ equals either $(-10)(5) = -50$

or $(- 2)(25) = -50$

Please proceed to question 18 below.

Question 18

Apply your knowledge to find the value of

$(-2p)^3$ when $p = -3$

(A) 216

(B) -216

(C) 54

(D) None of these.

V

80/1

The expression is the product of two parts. Let us take them one at a time.

(a) The meaning of m^2 is $m \cdot m$ which is $3 \cdot 3$, and it equals 9 .

(b) The value of $3m$ is $3 \cdot 3$, which is 9 .

Now, since

$$p = -6$$

$$-p = 6$$

Then

$$3m - p =$$

$$9 + 6 =$$

$$15$$

Thus, we have $9 (15)$

which is equal to 135

Then this choice is correct.

Please proceed to question 20 below.

80/2

Question 20

Apply your knowledge to evaluate

$$3x^2 + 2xy - y^2$$

when $x = -1$

and $y = -5$

(A) -18 (C) - 6

(B) -12 (D) 44

This problem calls for the DISTRIBUTIVE PROPERTY first, giving us:

$$3m(2m) + 3m(-5) - 4(3) - 4(-2m)$$

which equals $6m^2 - 15m - 12 + 8m$

and this equals $6m^2 - 7m - 12$

when we combine terms.

Therefore, this choice is correct.

Please proceed to question 16 below.

Question 16

Apply your knowledge to find the value of $5x^3$ when $x = -3$.

(A) - 45

(B) -135

(C) 135

(D) - 3375

V

You chose $\frac{b}{x} = a$

as the equation whose solution is $x = \frac{a}{b}$

We can check this by substituting the given solution in place of x in the equation you chose.

Your equation: $\frac{b}{x} = a$

or $b\frac{1}{x} = a$ (but $\frac{1}{x}$ is the inverse of x)

therefore,

Substituting: $b(\frac{b}{a}) \stackrel{?}{=} a$ ($\frac{1}{x}$ = the inverse of $\frac{a}{b} = \frac{b}{a}$)

As a check: $\frac{b^2}{a} \neq a$

Therefore, something is wrong.

Since the solution was " given " it can't be wrong. Since each step taken is legitimate, the conclusion must be that the original equation you chose must be wrong.

Please return to page 98/2 and try this question again.

You have divided the numbers in reverse order. Remember that

8 divided by 2 is 4 , but 2 divided by 8 is $\frac{1}{4}$.

This stated in symbols is:

$$\frac{a}{b} \neq \frac{b}{a} \quad \text{if} \quad \begin{matrix} a \neq 1 \\ b \neq 1 \end{matrix} \quad \text{and} \quad \begin{matrix} a \neq 0 \\ b \neq 0 \end{matrix}$$

Please return to page 106/2 and try question 4 again.

V

VOLUME 5 SEGMENT 3 Begins here:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48 and 50	2	3	(Sequence Number)
	54 and 56	0	4	(Type of Punch Card)
	60 and 62	0	5	(Volume Number)
	66 and 68	0	3	(Segment Number)

Your READING ASSIGNMENT for this Segment is pages 138 - 140 .

SUPPLEMENTARY NOTE:

In the two previous segments you investigated and applied the rules for the addition, subtraction, and multiplication of signed numbers. It will come as no surprise to you that we now propose to investigate the rules that apply to the division of signed numbers.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Apply your knowledge to find the solution of the equation

$$mx = p$$

(A) $x = p - m$

(B) $x = m - p$

(C) $x = \frac{m}{p}$

(D) $x = \frac{p}{m}$

V

84
1

You have used the DISTRIBUTIVE PROPERTY and the rules for signed numbers correctly.

Although there is a lot of detail in this problem, you made no errors. In detail the steps can be shown in this manner:

$$(-3x)(-5x + 2y - z) = -3x (-5x) - 3x (+2y) - 3x (-z)$$

$$= +15x^2 - 6xy + 3xz$$

Please proceed to question 14 below.

84
2

Question 14

Apply the appropriate rules to express as simply as possible:

$$5 (3a + r) - 4 (3a + r)$$

(A) $3a + 9r$

(B) $3a - 3r$

(C) $3a + r$

(D) $3a$

V

Since

$$(-2p)^3 \text{ means } (-2p)(-2p)(-2p)$$

all that is necessary is to substitute the value for p.

Then we get:

$$(-2p)(-2p)(-2p) = (+6)(+6)(+6)$$

$$= (36)(6)$$

$$= 216$$

Therefore, this choice is correct.

Please proceed to question 19 below.

Question 19

Apply the proper principles to evaluate

$$m^2(3m - p) \quad \text{when} \quad m = 3 \quad \text{and} \quad p = -6$$

(A) 27

(B) 18

(C) 135

(D) 90

$\frac{86}{1}$

Since we were told that the solution is

$$x = \frac{a}{b}$$

We can check that value in this equation. Substituting the value for x we have

$$ab = \frac{a}{b}$$

But this statement is true only when b has the value 1.

Therefore, this choice is not correct.

Please return to page $\frac{98}{2}$ and try question 2 again.

$\frac{86}{2}$

It is important to keep the rules for signs clearly in mind as you do these problems. However, you must not forget other details in the process.

What mistake of ordinary arithmetic have you made?

Please return to page $\frac{94}{2}$ and try question 5 again.

This problem is made up of many parts and offers many opportunities for error. Let us approach it very carefully.

$$3x^2 + 2xy - y^2 = 3(x)(x) + 2(x)(y) - (y)(y)$$

$$= 3(-1)(-1) + 2(-1)(-5) - (-5)(-5)$$

$$= 3(+1) + 2(+5) - (+25)$$

$$= 3 + 10 - 25$$

$$= 13 - 25$$

$$= -12$$

Then this choice is correct.

You have now finished Segment 2 . Hand in the PUNCH CARD.
You should have entered in your NOTEBOOK the three rules listed in your text near the top of page 135 . The second set of three rules are more general statements which can be learned later.

You should now be able to complete assignment 5 , examples 8 - 12 .

V

$\frac{88}{1}$

Something is wrong here. When you started, the right side of the equation had p ; that is , +p .

What have you done to make it a minus p ? In any case, it appears that you have forgotten the proper procedure for solving an equation. You should find it in your NOTEBOOK under Volume 3 .

Return to page $\frac{83}{2}$ and try question 1 again.

$\frac{88}{2}$

The problem asks that you multiply one number by the reciprocal of a second. That is equivalent to dividing the first by the second.

Note: $a \frac{1}{b} = a \div b$

Now if $\frac{1}{4} = \frac{1}{b}$

then $4 = b$

What operation did you perform? This choice is not correct.

Return to page $\frac{104}{2}$ and try question 6 again.

V

$\frac{89}{1}$

You have divided the numbers in reverse order. Remember that 8 divided by 2 is 4 , but 2 divided by 8 is $\frac{1}{4}$.

This stated in symbols is:

$$\frac{a}{b} \neq \frac{b}{a} \quad \text{if} \quad a \neq 1 \quad \text{and} \quad a \neq 0$$
$$b \neq 1 \qquad\qquad b \neq 0$$

There is another mistake which you have made in this problem. Did you notice that you are asked for the <u>absolute</u> value of the quotient?

Return to page $\frac{106}{2}$ and try question 4 again.

$\frac{89}{2}$

This problem calls for a multiplication of one number by the reciprocal of a second. It is equivalent to dividing the first number by the second. However, you appear mixed up about this.

If the reciprocal of the second number is $-\frac{1}{3}$, what is the second number? Did you divide by the <u>reciprocal</u> or by the second number?

The reciprocal of $(-\frac{a}{b})$ is $(-\frac{b}{a})$.

Return to page $\frac{108}{2}$ and try question 8 again.

V

$\frac{90}{1}$

The definition of reciprocal states that the product of any number and its reciprocal is 1, not that the reciprocal is 1 .

If you multiply 7 by 1 , the result is not 1 . Therefore, 1 is not the reciprocal of 7 .

Return to page $\frac{100}{2}$ and try question 9 again.

$\frac{90}{2}$

You were asked which number does NOT have a reciprocal.

Since 1 times 1 equals 1 , 1 is its own reciprocal.

Then this choice is not correct.

Return to page $\frac{111}{2}$ and try question 11 again.

Since your text defines the division $\frac{a}{b}$ as the quantity which when multiplied by b gives a, it follows that this choice is correct.

Approaching this as the solution of an equation, we note that this choice offers an equation in which x is multiplied by b. Therefore, in order to solve the equation, it is proper to divide both sides of the equation by b, giving the value as listed in the problem,

$$x = \frac{a}{b}$$

Please proceed to question 3 below.

Question 3

Which expression do you recognize as equivalent to

$$(5x)(\frac{1}{3s})$$

(A) $\frac{5x}{15xs}$ (C) $15xs$

(B) $5x \div 3s$ (D) None of these.

The problem asks that you multiply one number by the reciprocal of a second. That is equivalent to dividing the first by the second; that is, dividing

$$-8 \quad \text{by} \quad 4$$

Then, applying the rules for signs, the result is

$$-2$$

And this choice is correct.

Note:

$$\boxed{(-a) \cdot \left(\frac{1}{b}\right) = -\left(\frac{a}{b}\right)}$$

Please proceed to question 7 below.

Question 7

Choose the value of

$$(+6) \div (+18)$$

(A) $+\frac{1}{3}$ (C) $-\frac{1}{3}$

(B) $+3$ (D) -3

It appears that you have used a proper process for solving the equation. However, there was some error in the way you used it. This choice shows a division <u>by</u> p on the right side.

Did you also divide by p on the left side? If you did, you would not have x remaining alone.

Please return to page 83/2 and try question 1 again.

Since the product of -2 and your choice is not 1, they are not reciprocals.

Please return to page 105/2 and try question 10 again.

V

$\frac{94}{1}$

Since the absolute value of a quotient equals the quotient of the absolute values, we merely divide 15 by 3 .

Then this choice is correct.

$$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$$

Please proceed to question 5 below.

$\frac{94}{2}$

Question 15

Choose the value of the quotient

$$(12) \div (-3)$$

(A) $-\frac{1}{4}$

(B) $\frac{1}{4}$

(C) 4

(D) - 4

You should have used the rule:

$$(-a)(-\frac{1}{b}) = +\frac{a}{b}$$

There was one detail which you overlooked. Can you locate it?

Return to page $\frac{108}{2}$ and try question 8 again.

The product of $\frac{2}{3}$ and $\frac{3}{2}$ is 1 .

Then these numbers are reciprocals, and this choice is not correct.

The reciprocal of

$$\frac{a}{b} \quad \text{is} \quad \frac{b}{a}$$

Return to page $\frac{111}{2}$ and try question 11 again.

V

96/1

This problem calls for the multiplication of $5x$ by the reciprocal of $3s$. The definition of division states that the quotient of two quantities is equal to the product of the first by the reciprocal of the second.

This rule, of course, is symmetric. Therefore, it can be stated as it applies here. The product of a quantity and the reciprocal of another quantity is equal to the quotient of the two quantities. But you do not have the quotient in this choice; therefore, it is not correct.

Return to page 91/2 and try question 3 again.

96/2

There is a fact about reciprocals which you have missed. Any number and its reciprocal always have the same sign. Since the given number was positive, and this choice is negative, it is not correct.

Return to page 100/2 and try question 9 again.

It appears that you have changed the order of division. Remember, the COMMUTATIVE PROPERTY does not apply to division. Therefore, you must not change the order of the numbers.

$$a \div b \neq \frac{b}{a}$$

Return to page $\frac{92}{2}$ and try question 7 again.

The product of

$$-\frac{2}{3} \quad \text{by} \quad -\frac{2}{3}$$

is positive, but it is not 1 . Therefore, they are not reciprocals, and this choice is not correct.

Return to page $\frac{116}{2}$ and try question 12 again.

Dividing both sides of the given equation by m we get x alone on the left side, and right side becomes

$$\frac{p}{m}$$

Therefore, this choice is correct.

It is also important to realize that your text has defined division in terms of a solution to an equation such as this.

Please proceed to question 2 below.

Question 2

Apply your knowledge to find the equation whose solution is

$$x = \frac{a}{b}$$

(A) $bx = a$

(B) $ax = b$

(C) $\frac{b}{x} = a$

(D) $ab = x$

It is quite simple to say that any result in algebra is either positive or negative, and that you cannot be very far wrong if you have the sign wrong. Obviously this is nonsense; you can be right if you learn the rules and apply them. Otherwise, you are just guessing.

You might note that it is possible to state the rules for signs in dividing two numbers in this form:

(1) If the numbers have the same sign, the result is plus.

(2) If the numbers have different signs, the result is minus.

Return to page 94/2 and try question 5 again.

As your text has stated, the reciprocal of a product is the product of the reciprocals. Where have you slipped up?

This choice is not correct.

Return to page 109/2 and try question 13 again.

$\frac{100}{1}$

Since this problem calls for the multiplication of -2 by the reciprocal of -3, it is equivalent to the division of -2 by -3. Then the result is positive, and the quotient of 2 divided by 3 is $\frac{2}{3}$.

This is an example of the rule:

$$(-a)(-\frac{1}{b}) = +\frac{a}{b}$$

Please proceed to question 9 below

$\frac{100}{2}$

Question 9

Which choice do you recognize as the reciprocal of 7 ?

(A) 1

(B) $\frac{1}{7}$

(C) $-\frac{1}{7}$

(D) -7

If you will go over this problem more carefully, you will find that one of the other choices is correct.

Therefore, this choice is not correct.

Please return to page $\frac{91}{2}$ and try question 3 again.

The product of $-\frac{5}{4}$ and $-\frac{4}{5}$ is $+1$

Then these numbers are reciprocals, and this choice is not correct.

The reciprocal of $-\frac{a}{b}$ is $-\frac{b}{a}$

Please return to page $\frac{111}{2}$ and try question 11 again.

102
1

You seem to be quite confused. This problem called for a division. Have you looked up the rules for division?

Please do so before continuing.

Please return to page 92/2 and try question 7 again.

102
2

That's very close, but you do not have the correct choice.

Have you forgotten that you are dealing with signed numbers?

Please return to page 114/2 and try question 14 again.

V

The reciprocal of a negative number is also negative.

Therefore, this choice is not correct regardless of the product of the absolute values.

Please return to page 116/2 and try question 12 again.

You should multiply by the reciprocal of the divisor. Which is the divisor? Did you use its sign? Did you use the correct value for the reciprocal of the divisor? Finally did you multiply correctly? If you did each of these correctly and used the rules for signs properly, you would not get this choice.

Please return to page 119/2 and try question 15 again.

The reciprocal of -3 is $-\frac{1}{3}$. Then this example is equivalent to multiplying 12 by $-\frac{1}{3}$. Since one number is positive and one is negative, the product is negative. Also the product of

$$12 \text{ and } \frac{1}{3} \text{ is } 4$$

Then the result is -4, and this choice is correct.

You might note that the rules for signs in division can be put in this form:

(1) If the two numbers have the same sign, the result is positive.

(2) If the two numbers have different signs, the result is negative.

Do you notice that the rules for signs in division are the same as the rules for signs in multiplication?

Please proceed to question 6 below.

Question 6

Choose the value of the expression

$$\left(-8\right)\left(\frac{1}{4}\right)$$

(A) -32

(B) $-\frac{1}{2}$

(C) -2

(D) $-\frac{1}{32}$

Since the product of

$$7 \text{ and } \frac{1}{7} \text{ is } 1$$

they are reciprocals.

$a \cdot \frac{1}{a} = 1$
the reciprocal of a is $\frac{1}{a}$
the reciprocal of $\frac{1}{a}$ is a

Please proceed to question 10 below.

Question 10

Which choice do you recognize as the reciprocal of -2 ?

(A) -1

(B) $\frac{-1}{-2}$

(C) $-\frac{1}{2}$

(D) $+\frac{1}{2}$

106
1

This problem calls for the multiplication of $5x$ by the reciprocal of $3s$. Then, according to the definition of division, it is equivalent to the quotient of these quantities.

This can be stated

in symbolic form:

$$\text{if } (a) \div (b) = (a) \times \left(\frac{1}{b}\right)$$

then by the SYMMETRIC PROPERTY

$$(a) \times \left(\frac{1}{b}\right) = (a) \div (b)$$

Then this choice is correct.

Please proceed to question 4 below.

106
2

Question 4

Choose the absolute value of the quotient

$$(-15) \div (3)$$

(A) -5

(B) 5

(C) $\frac{1}{5}$

(D) $-\frac{1}{5}$

V

It appears that there are too many details for you to watch at one time. It is necessary to apply the rules for signed numbers first. Then, after you have determined the sign of the result, you must do the arithmetic correctly. If either part is wrong, your result is wrong; never tell yourself that you are half right.

What should be the sign of the result when you divide a positive number by a negative number? Since division by a number is equivalent to multiplication by its reciprocal, which is the divisor and what is its reciprocal.

Work carefully and slowly and follow the rules.

Please return to page 114/2 and try question 14 again.

There are two parts to the calculation of this problem; one is getting the signs correct, and the other is getting the arithmetic correct. Remember that

$$\frac{xy^2}{5m} = \frac{x \cdot y \cdot y}{5 \cdot m}$$

If you substitute the values for each of the letters and do the arithmetic, watching the signs, you will find that this choice is not correct.

Please return to page 123/2 and try question 17 again.

V

This problem can be done as an ordinary arithmetic calculation. However, it becomes easier if you apply the definition of division from your text.

$$a \div b = a \cdot \frac{1}{b}$$

$$a \cdot \frac{1}{b} = \frac{a}{b}$$

We have:

$$(+6) \div (+18) = (+6) \cdot (+\tfrac{1}{18})$$
$$= +\frac{6}{18}$$
$$= +\frac{1}{3}$$

Therefore, this choice is correct.

Please proceed to question 8 below.

Question 8

Choose the value of

$$(-2)\ (-\tfrac{1}{3})$$

(A) $+6$

(B) -6

(C) $+\frac{2}{3}$

(D) $-\frac{2}{3}$

Since the product of

$$-\frac{2}{3} \text{ and } -\frac{3}{2} \text{ is } +1$$

These numbers are reciprocals. Then this choice is correct.

You should note that the reciprocal of a fraction is always the fraction inverted. Of course, the sign of the reciprocal must agree with the sign of the number.

Please proceed to question 13 below.

Question 13

Choose the reciprocal of

$$(-5)\ \left(\frac{7}{9}\right)$$

(A) $(-5)\ \left(\frac{9}{7}\right)$

(B) $\left(-\frac{1}{5}\right)\ \left(\frac{9}{7}\right)$

(C) $\left(\frac{1}{5}\right)\ \left(\frac{9}{7}\right)$

(D) $\left(-\frac{1}{5}\right)\ \left(\frac{7}{9}\right)$

Remember that division is defined as multiplication by the reciprocal of the divisor. Then we have:

$$\left(-12x\right) \div \left(-\frac{1}{2}\right) = \left(-12x\right) \cdot \left(-\frac{2}{1}\right)$$

$$= \left(-\frac{12x}{1}\right) \cdot \left(-\frac{2}{1}\right) \quad [\, -a \cdot -b = +ab \,]$$

$$= +\frac{(12x)\,(2)}{(1)\,(1)}$$

$$= +\frac{24x}{1}$$

$$= 24x$$

Then this choice is correct.

Please proceed to question 16 below.

Question 16

Apply your knowledge to find the expression equivalent to

$$\frac{15x^2 - 18x + 9}{-3}$$

(A) $-5x^2 - 18x + 9$

(B) $-5x^2 + 6x + 9$

(C) $-5x^2 + 6x + 3$

(D) $-5x^2 + 6x - 3$

The product of 2 and $\frac{1}{2}$ is 1 ; therefore, the product of -2 and $-\frac{1}{2}$ is also 1 .

Then these are reciprocals.

The definition that applies is:

if	a	•	b	=	1
then	a	is the reciprocal of			b
and	b	is the reciprocal of			a

Please proceed to question 11 below.

Question 11

Choose the number which does NOT have a reciprocal.

(A) 1

(B) $\frac{2}{3}$

(C) $-\frac{5}{4}$

(D) 0

There are two parts to the calculation of this problem; getting the signs correct, and doing the arithmetic correctly.

Since $x = 2$

$y = -10$

and $m = -4$ We have:

$$\frac{xy^2}{5m} = \frac{x \cdot y \cdot y}{5 \cdot m}$$

$$= \frac{(2)\ (-10)\ (-10)}{5\ (-4)}$$

$$= \frac{(\ 2\ )\ (+100)}{-20}$$

$$= \frac{200}{-20}$$

$$= -10$$

Then this choice is correct.

Please proceed to question 18 below.

Question 18

Apply your knowledge to evaluate

$$\frac{2x - 3x^2}{4x} \quad \text{when} \quad x = -2$$

(A) 2

(B) -1

(C) 5

(D) -4

V

When substituting, do one step at a time.

It appears that you have made an error in the value of

$$2a - b$$

It can be written as

$$(2)(a) - (1)(b)$$

Now, if you substitute the values for a and b and are very careful with the signed numbers, you should find that this choice is not correct.

Please return to page 120/2 and try question 20 again.

Since this equation states that subtracting 3 from x gives the value of y , it is impossible for y to be larger than x .

Please return to page 135/2 and try question 1 again.

As your text has stated, the reciprocal of a product is the product of the reciprocals. Since each factor of this choice is the reciprocal of one factor of the original expression, this choice is correct.

Please proceed to question 14 below.

Question 14

Apply your knowledge to find the value of

$$\left(\frac{21}{5}\right) \div (-7)$$

(A) $-\frac{3}{5}$

(B) $\frac{3}{5}$

(C) $-\frac{5}{3}$

(D) $\frac{5}{3}$

Careful substitution is a must; then accurate interpretation of the exponents is required.

It is important to realize that signs must be correct before you worry about the arithmetic. If you make a mistake in signs, you may do the wrong arithmetic. For example, 7 added to negative 9 gives a different result from 7 added to positive 9 .

You must have signs correct in order to proceed. What error have you made?

Return to page 129/2 and try question 19 again.

Adding 5 to the right side of the given equation gives

$$x + 6$$

However, adding 5 to the left side of the original equation does not give

$$3x + 5$$

What is the result of adding 5 to the left side of the equation?

Return to page 128/2 and try this question again.

V

116
1

Since any number multiplied by zero gives zero as a result, there is no number which can be multiplied by zero to give 1 .

Then zero has no reciprocal, and this choice is correct.

It is important to note that zero is the only number which does not have a reciprocal.

Please proceed to question 12 below.

116
2

Question 12

Which choice do you recognize as the reciprocal of $-\frac{2}{3}$?

(A) $-\frac{2}{3}$

(B) $\frac{2}{3}$

(C) $-\frac{3}{2}$

(D) $\frac{3}{2}$

V

The DISTRIBUTIVE PORPERTY must be used when an expression containing more than one term is divided by any quantity. Did you apply the DISTRIBUTIVE PROPERTY correctly?

This choice is not correct.

Note: $$\frac{a + b + c}{d} = \frac{a}{d} + \frac{b}{d} + \frac{c}{d}$$

Return to page 110/2 and try question 16 again.

Since the original equation had $3x$ in it, (that is, 3 times x) the only way to remove the 3 is to employ the counter operation and divide by 3 . However, there is no division by 3 on the right hand side of the equation.

Then this chcice is not correct.

Return to page 138/2 and try question 3 again.

118
1

Since

$$\frac{2x - 3x^2}{4x} = \frac{(2)(x) - 3(x)(x)}{(4)(x)}$$

it is only necessary to substitute the given values and do the calculation. If you are very careful, you will discover that this choice is not correct.

Please return to page 112/2 and try question 18 again.

118
2

According to this equation, it is true that y is smaller than x . But $y < x$ would give that information. What additional information does the equation give? Since this choice does not give the best meaning, it is not correct.

Please return to page 135/2 and try question 1 again.

Since division by a number is equivalent to multiplication by its reciprocal, we have:

$$\left(\frac{21}{5}\right) \div (-7) = \left(\frac{21}{5}\right) \cdot \left(-\frac{1}{7}\right)$$

$$= -\frac{(21)\ (1)}{(5)\ (7)}$$

$$= -\frac{(3)\ (1)}{(5)\ (1)}$$

$$= -\frac{3}{5}$$

Therefore, this choice is correct.

Please proceed to question 15 below.

Question 15

Apply your knowledge to find the expression equivalent to

$$(-12x) \div \left(-\frac{1}{2}\right)$$

(A) 24x

(B) 6x

(C) -24x

(D) - 6x

The calculation should look like this:

Given: $p = 2$ [by substitution]

$$\frac{(2 - 3p)^3}{4p^2} = \frac{(2 - 3.2)^3}{4(2)(2)}$$ [multiplication precedes subtraction]

$$= \frac{(2 - 6)^3}{4(4)}$$ [subtraction]

$$= \frac{(-4)^3}{16}$$ [meaning of exponent 3]

$$= \frac{(-4)[(-4)(-4)]}{16}$$ [ASSOCIATIVE LAW]

$$= \frac{(-4)(+16)}{16}$$ [$\frac{ab}{b} = a$]

$$= -4$$

Then this choice is correct.

Please proceed to question 20 below.

Question 20

Apply your knowledge to evaluate

$$\frac{5a - (2a - b)}{3b + 1}$$ when $a = 6$ and $b = -3$

(A) $-\frac{15}{8}$ (C) $-\frac{3}{2}$

(B) $-\frac{21}{8}$ (D) $-\frac{21}{10}$

If you subtract 5 from the right side of the given equation, the result is

$$x - 4$$

However, if 5 was subtracted from the left side of the equation, you would get

$$3x - 10$$

but you thought you would get

$$3x$$

Then this choice is not correct.

Please return to page 128/2 and try question 2 again.

Whatever operation has been performed on one side of the equation must be performed on the other.

Examining the original equation and this choice, we find that the left side has been multiplied by 2 . But the right side has not been multiplied by 2 . Therefore, this choice is not correct.

Please return to page 141/2 and try question 5 again.

V

122/1

Remember, in order to remove a term from one side of an equation, we must choose the opposite operation and apply that operation to both sides of the equation.

In this choice, we find that the $+2x$ which was on the right side of the equation is no longer there. Then it must have been subtracted from both sides. But we have $7x$ on the left side, so that $2x$ was not subtracted.

Then this choice is not correct.

Please return to page 130/2 and try question 4 again.

122/2

Since the equation has the variable x with a coefficient of $\frac{2}{5}$, the proper procedure in solving the equation involves multiplying by 5 and dividing by 2. Of course this is done to both sides of the equation after getting the term containing x alone on one side.

There are three separate operations to be performed. If you follow this procedure, you will find that this choice is not correct.

Please return to page 151/2 and try question 7 again.

The DISTRIBUTIVE PROPERTY must be used when dividing an expression containing more than one term by any quantity. Then the work should be:

$$\frac{15x^2 - 18x + 9}{-3} = \frac{15x^2}{-3} - \frac{18x}{-3} + \frac{9}{-3}$$

$$= -5x^2 + 6x - 3$$

Then this choice is correct.

Please proceed to question 17 below.

Question 17

Apply your knowledge to find the value of

$$\frac{xy^2}{5m}$$ when $x = 2$, $y = -10$, and $m = -4$

(A) 10

(B) -10

(C) 20

(D) -20

124
1

You did not consider the other choices carefully enough. One of the other choices is correct.

Please return to page 135/2 and try question 1 again.

124
2

The original equation had the term +p on the left side. In order to remove it, it is necessary to use the counter operation and subtract p from both sides of the equation. Since this choice does not show a subtraction of p on the right side of the equation, it is not correct.

Please return to page 138/2 and try question 3 again.

You should know that

$$-3x^2 = -3(x)(x)$$

not

$$(-3x)(-3x)$$

It appears that you have confused them, and perhaps you have made other errors since this choice is not correct.

Return to page 112/2 and try question 18 again.

If we divide both sides of the given equation by 3 we get

$$x + 2$$

on the left side. But we must also divide on the right side. Since that has not been done, this choice is not correct.

Return to page 146/2 and try question 6 agian.

V

126
1

When substituting, do one step at a time.

Did you make an error in calculating the value of

$$3b + 1$$

Watch for an error in signs, which gives you this incorrect choice.

Return to page 120/2 and try question 20 again.

126
2

At first glance it appears that both sides of this equation have been multiplied by 3 . That would be a permissible operation and would yield an equivalent equation. However, what went wrong in the multiplication? The DISTRIBUTIVE PROPERTY is one we will meet again and again.

$$a (b + c) = ab + ac$$

Return to page 141/2 and try this question again.

Remember, in order to remove a term from one side of an equation, we must choose the opposite operation and apply that operation to both sides of the equation.

In the original equation, the term -8 appeared on the left side. Since this choice shows that it is no longer there, 8 must have been added to both sides, but we can see that 8 was <u>not</u> added on the right side. Then this choice is not correct.

Please return to page 130/2 and try question 4 again.

In solving this equation, it is necessary to get all x terms on one side of the equation. This can be done by adding x to both sides. Next we must remove the $+1$ from the left side. What process of mathematics can remove the 1 which has been added on?

Finally, it is necessary to divide both sides of the equation by the coefficient of x. If you go through each of these steps carefully, you will find that this choice is not correct.

Please return to page 137/2 and try question 8 again.

V

128
1

It is important to recognize that an equation states that two quantities are equal. Since the quantity on one side is merely y, and that on the other side is 3 less than x, it follows that the meaning of the equation is that y is equal to 3 less than x.

Please proceed to question 2 below.

128
2

Question 2

Choose the derived equation which has the same solution set as the equation

$$3x - 5 = x + 1$$

(A) $3x = x + 6$

(B) $3x + 5 = x + 6$

(C) $3x = x - 4$

(D) $4x - 5 = 1$

$$\frac{2x - 3x^2}{4x} = \frac{(2)(x) - 3(x)(x)}{(4)(x)}$$

$$= \frac{(2)(-2) - 3(-2)(-2)}{(4)(-2)}$$

$$= \frac{-4 - 3(+4)}{-8}$$

$$= \frac{-4 \quad -12}{-8}$$

$$= \frac{-16}{-8}$$

$$= +2$$

Then this choice is correct.

Please proceed to question 19 below.

Question 19

Apply your knowledge to evaluate

$$\frac{(2 - 3p)^3}{4p^2} \qquad \text{when } p = 2$$

(A) 1

(B) -1

(C) 4

(D) -4

V

130
1

In the original equation there was a term $+p$ on the left side. If we wish to remove it, we must use the counter operation and subtract p from both sides. This would give us:

$$3x + p = 7w \quad \swarrow -p$$

$$3x = 7w - p$$

Then this choice is correct.

Please proceed to question 4 below.

130
2

Question 4

If you apply the proper principles to transform the equation

$$5x - 8 = 10 + 2x$$

which equation would be the result after two steps?

(A) $7x - 8 = 10$

(B) $5x = 2 + 2x$

(C) $3x = 18$

(D) None of these

V

Let us first consider the value of

$$2a - b$$

which equals

$$(2)(a) - (1)(b) = (2)(6) - (1)(-3)$$

$$= 12 + 3$$

$$= 15$$

Now, we have:

$$\frac{5a - (2a - b)}{3b + 1} = \frac{30 - 15}{3(-3) + 1}$$

$$= \frac{15}{-9 + 1}$$

$$= \frac{15}{-8}$$

$$= -\frac{15}{8}$$

And this choice is correct.

You have now finished this Segment. Hand in the PUNCH CARD.

You should have enetered in your NOTEBOOK the following rules:

(1) When dividing two numbers which have the same sign, the result is positive.

(2) When dividing numbers which have different signs, the result is negative.

(3) $\frac{a}{b}$ and $\frac{b}{a}$ are reciprocals of each other.

(4) Reciprocals have the same sign.

You should now be able to complete assignment 5 , problems 9 - 12 .

132
1

Let us consider the original equation

$$5x - 2 = 7$$

We proceed as follows:

$5x - 2 = 7$	$\times +2$		(Adding 2 to both sides)
$5x = 9$	$\times (2)$		(Doubling both sides)
$10x = 18$			

Since this disagrees with this choice, the choice is not correct. Note that any correct operations must yield an equivalent equation; we performed those operations that would make the left side of our equation agree with the left side of this choice. But the right side should also have agreed.

Please return to page 141/2 and try this question again.

132
2

In order to get x alone on one side of the equation, we must add a to both sides. But, when you do that, we do not get a + b on the right side.

Then this choice is not correct.

Please return to page 142/2 and try question 9 again.

If you add x to the left side of the given equation, the result is

$$4x - 5$$

However, if you add x to the right side of the given equation, the result would be

$$2x + 1$$

What operation was performed on the right side of the equation to give this choice that you picked?

Remember that you must perform the same operation on both sides of the equation.

Please return to page 128/2 and try question 2 again.

If we subtract 6 from both sides of the equation, we do get 4 on the right side. But we do not get $1x$ on the left side.

Therefore, this choice is not correct.

Please return to page 146/2 and try question 6 again.

134
1

Since it is y you are to solve for, you must remove the 3m from the left side of the equation. What process of mathematics will accomplish this for you? You should find that this choice is not correct.

Please return to page 157/2 and try question 10 again.

134
2

You were right in deciding to remove the 2 attached to the t on the left side of the equation. What process did you use to remove it? Did you think you were dividing? Perhaps you should review the DISTRIBUTIVE PROPERTY, which should be applied in this problem.

$$\frac{a + b}{c} = \frac{a}{c} + \frac{b}{c}$$

Please return to page 149/2 and try question 12 again.

VOLUME 5 SEGMENT 4 BEGINS HERE:

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2	4 (Sequence Number)
	54	and	56	0	4 (Type of Punch Card)
	60	and	62	0	5 (Volume Number)
	66	and	68	0	4 (Segment Number)

Your READING ASSIGNMENT for this Segment is pages 157 - 158 .

SUPPLEMENTARY NOTE:

In the process of transforming an equation, you will always be applying one of the axioms. When you have had a good bit of experience with the process, you will do many steps mentally. However, until you are familiar with the work, you should write down every step of the work. Even after you arrive at the point of doing certain steps in your head, you must always be able to state the axiom which you are using at that point.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

Which do you recognize as the best meaning for the open sentence

$$y = x - 3$$

(A) y is larger than x

(B) y is smaller than x

(C) The value of y is equal to 3 less than the value of x

(D) None of these.

V

136/1

Since the object in solving an equation is to isolate the variable, we can see that part of the task in this problem is to remove the $2x$ from the right side of the equation and to remove the -8 from the other side. Choose the opposite operation in each case.

Take another look at the other choices. One of them is an equation equivalent to the original.

Please return to page 130/2 and try question 4 again.

136/2

In order to solve for x, it is necessary to remove the w from the left side of the equation. What operation of mathematics has attached the w? Then the inverse operation can be used to remove it.

The process you used was wrong since this choice is not correct.

In the form $\frac{x}{a} = b$ we use the operation $\times a$ to isolate x.

Please return to page 144/2 and try question 11 again.

V

The procedure for solving this equation is:

$\frac{2}{5}x + 8 = 0 \quad \prec -8$ (Subtract 8 from both sides)

$\frac{2}{5}x = -8 \quad \prec -5$ (Multiply by 5 on both sides)

$2x = -40 \quad \prec \div 2$ (Divide by 2 on both sides)

$x = -20$

Then this choice is correct.

Please proceed to question 8 below.

Question 8

Apply your knowledge to solve the equation

$$3x + 1 = 7 - x$$

Which statement about the value of x is true?

(A) $x \in \{1 , 3 , 5\}$

(B) $x \in \{2 , 4 , 6\}$

(C) $1 < x < 5$

(D) $2 < x < 6$

138
1

If we add 5 to each side of the given equation, this is the result. Therefore, this equation is equivalent to the given equation. It is called a derived equation.

This choice is correct.

Please proceed to question 3 below.

138
2

Question 3

In the process of solving for x, choose the derived equation equivalent to the equation

$$3x + p = 7w$$

(A) $x = 7w - 3p$

(B) $3x = 7w - p$

(C) $3x = 7w + p$

(D) $x = \frac{7w + p}{3}$

If we subtract 6 from both sides of the equation, we do get 3x on the left side. But we do not get 16 on the right side.

Then this choice is not correct.

Please return to page 146/2 and try question 6 again.

Is this the value you got for y ? Or did you slip up on a sign along the way?

Remember it is +y which you want to get alone on one side of the equation.

Please return to page 157/2 and try question 10 again.

V

140/1

The fundamental rule in transforming equations is to apply whatever operation you choose to BOTH sides of the equation.

From the right side of the equation in this choice, it appears that you have subtracted 2 . How was the 2 attached on the left side of the original equation?

Then subtraction could not remove it, and this choice is not correct.

Please return to page 149/2 and try question 12 again.

140/2

You should remember that two terms can be combined only when they are like terms; that is, when their letter parts are identical.

Thus,

$$m + 3m = 4m$$

and

$$my + 3my = 4my$$

But it is not possible to combine

$$m + 3my$$

It appears that you made an error of this sort.

Please return to page 164/2 and try question 14 again.

The work in this problem could be shown as follows:

$5x - 8 = 10 + 2x$ $\swarrow +8$ (Adding 8 to both sides)

$5x = 18 + 2x$ $\swarrow -2x$ (Subtracting 2x from both sides)

$3x = 18$

Then this choice is correct.

Please proceed to question 5 below.

Question 5

In the process of finding the solution set for x , choose the derived equation which is equivalent to

$$5x - 2 = 7$$

(A) $10x - 4 = 7$

(B) $15x - 2 = 21$

(C) $10x = 10$

(D) None of these

142
1

The solution of this equation is as follows:

$3x + 1 = 7 - x \quad +x$ (Add x to both sides)

$4x + 1 = 7 \quad -1$ (Subtract 1 from each side)

$4x = 6 \quad \div 4$ (Divide both sides by 4)

$x = 1.5$

Since 1.5 is between 1 and 5, but does not belong in any of the other sets offered, this choice is correct.

Please proceed to question 9 below.

142
2

Solve for x and choose the solution of the equation

$$x - a = 2a + b$$

(A) $x = 3a + b$

(B) $x = a + b$

(C) $x = 2a + b + a$

(D) None of these.

You have almost solved the equation correctly. When you transformed the equation into a form having "prt" on one side, what was the expression on the other side?

Is it possible that you had a mistake in sign at that point?

Please return to page 159/2 and try question 13 again.

It is true that the fraction $\frac{2}{3}$ is a nuisance, but did you have a mathematical procedure for getting rid of it?

This choice is not correct.

Please return to page 150/2 and try question 15 again.

V

144
1

Since it is y we must solve for, we must remove the $3m$ from the left side of the equation. But the $3m$ is attached on the left side by addition. Therefore, it is necessary to subtract the $3m$ in order to remove it. Subtracting $3m$ from both sides, we get

$$y = 7 - p - 3m$$

Then this choice is correct.

Please proceed to question 11 below.

144
2

Solve for the variable x and choose the solution of the equation

$$\frac{x}{w} = 5s$$

(A) $x = w + 5s$

(B) $x = w - 5s$

(C) $x = \frac{5s}{w}$

(D) $x = 5sw$

The letter k was in the original equation. Since the only mathematical operations which can cause it to vanish

$$k - k = 0 \quad \text{or} \quad k \div k = 1$$

did not occur in the solving of the equation, it should be in the solution.

Did you attempt to perform the division shown in the original equation? Remember that the DISTRIBUTIVE PROPERTY says that each term in the expression

$$p - k$$

must be divided by $5k$.

i. e.

$$\frac{p - k}{5k} = \frac{p}{5k} - \frac{k}{5k} = \frac{p}{5k} - \frac{1}{5}$$

Although the k disappeared in the second fraction, it still remains in the first.

Please return to page 164/2 and try question 14 again.

Let us give you some help. Always apply the DISTRIBUTIVE PROPERTY first if you see the classic arrangement of terms in the

$$a (b + c)$$

form. Then collect like terms on each side of the equation wherever possible.

Now in this problem, which is an incomplete problem (where you haven't even been told what to solve for), look at the various choices presented and decide what operation on your derived equation can be used to produce one of the choices. When you verify that, you will have answered this problem correctly.

Please return to page 163/2 and tackle this problem again.

V

146
1

If we examine the other choices, one by one, we find that they do not offer equations equivalent to the original.

Then this choice must be correct.

Please proceed to question 6 below.

146
2

Apply the principles for transforming equations and choose the equation equivalent to

$$3x + 6 = 10$$

(A) $x + 2 = 10$

(B) $x + 2 = \frac{10}{3}$

(C) $x = 4$

(D) $3x = 16$

In order to get x alone on one side of the equation, we must add a to both sides. Although it appears that you started to do that, you never finished the job.

What can you do to simplify your result? If you gave the answer

$$2 + 3$$

when the result should be 5, we would not accept it. You are supposed to complete the calculation.

Please return to page 142/2 and try question 9 again.

The letter p appeared in the original equation. What happened to it? It must appear in the solution, since the only mathematical operations that would remove it did not properly apply:

$$p - p = 0$$

and

$$\frac{p}{p} = 1$$

Please return to page 159/2 and try question 13 again.

V

148/1

It is important to simplify

$$5 - (1 - x)$$

correctly. It can be considered as

$$5 - 1(1 - x)$$

but it is certainly not the same as

$$-5(1 - x)$$

When simplified, the right - hand side of the equation should be

$$5 - 1 + x$$

This choice is not the correct answer for the problem.

Please return to page 167/2 and try question 16 again.

148/2

In solving this equation, it is necessary to apply the DISTRIBUTIVE PROPERTY first. Next you should combine similar terms on each side of the equation. Then you must get only x terms on one side, and only number terms on the other side. Be sure that you are using a process of mathematics for each step. If you do a careful and correct job, you will find that this choice is not correct.

Please return to page 156/2 and try question 18 again.

In order to solve for x it is necessary to remove the w from the left side of the equation. Since the w was attached to the x by division, it can be removed by multiplying by w. Of course, we must multiply by w on both sides of the equation. Then we get

$$x = 5sw$$

and this choice is correct.

Please proceed to question 12 below.

Question 12

Solve for t and choose the solution of the equation

$$2t = p - 2w$$

(A) $t = p - w$

(B) $t = p - 2w - 2$

(C) $t = \frac{p - 2w}{2}$

(D) None of these

150
1

Since we wish to solve for p we should get the term containing p alone on one side of the equation. Then our first job is to get rid of the division by 5K.

$n = \frac{p - k}{5K}$ $\times(5k)$ (Multiply both sides by 5k)

$5kn = p - k$ $+k$ (Add k to both sides)

$5kn + k = p$ [(Use Symmetric Principle)

$p = 5kn + k$

Therefore, this choice is correct.

Please proceed to question 15 below.

150
2

Question 15

Apply the appropriate principles to solve the equation

$$T = \frac{2}{3}(P - y)$$

for y .

(A) $y = P - T$

(B) $y = \frac{3}{2}T - P$

(C) $y = 2P - 3T$

(D) $y = P - \frac{3}{2}T$

V

Dividing the left side of the equation by 3 , we would have

$$x + 2$$

But then it is necessary to divide the right side by 3 also.

When we do this the result is

$$\frac{10}{3}$$

Then we have the equation

$$x + 2 = \frac{10}{3}$$

and this choice is correct.

Notice that it would not be reasonable to do this as a step in the solution of the equation. However, the question did not ask for the solution of the equation; merely for an equation equivalent to the original, which we did find.

Please proceed to question 7 below.

Question 7

Apply the principles of operations in equations to solve the equation

$$\frac{2}{5} x + 8 = 0$$

Which statement about the value of x is true?

(A) $x \in \{\text{odd integers}\}$

(B) $x \in \{\text{positive integers}\}$

(C) $x \in \{\text{fractions}\}$

(D) $x \in \{\text{negative numbers}\}$

152
1

Solving the equation means transforming it into one which has only the variable x alone on one side. What operation of mathematics can you perform to accomplish this? On reconsideration of this problem, you will find that one of the other choices is correct.

Please return to page 142/2 and try question 9 again.

152
2

What do you get when you combine the terms

$$p + prt$$

The correct answer is that they cannot be combined since the letters are not exactly the same. Note that

$$p + p = 2p$$

and that $prt + prt = 2prt$

Since you had an error in algebra, this choice is not correct.

Please return to page 159/2 and try question 13 again.

It is important to simplify

$$5 - (1 - x)$$

correctly. It can be considered as

$$5 - 1(1 - x)$$

which when simplified gives

$$5 - 1 + x$$

Then this choice is not the correct answer for the problem.

Please return to page 167/2 and try question 16 again.

Remember, directed numbers can be considered as points on the NUMBER LINE. There are just two conditions possible; namely, the two points are located at the same position on the line (when they are equal) or one point is located to the left of the other (when they are unequal). It is impossible to have the same two numbers both equal and unequal at the same time.

Therefore, this choice is incorrect.

Please return to page 168/2 and try this question again.

V

154/1

If you will examine the equation, you will discover that solving for t requires only that the coefficient 2 be removed.

What process of mathematics will remove it? You will discover when you complete the solution that one of the other choices is correct.

Please return to page 149/2 and try question 12 again.

154/2

If you follow proper procedures for each step of the solution, you will find that this choice is not correct.

There are many orders of steps that are possible. Here is one sequence of steps that will produce the answer.

$\propto \cdot 3, \quad [\ D\ , \quad \propto -2n\ , \quad \propto -27\ , \quad \propto \div 13$

Please return to page 161/2 and try question 19 again.

Very close, but not quite correct. Did you arrange to get y alone on one side of the equation; that is , positive ly ?

This choice is not correct.

Return to page 150/2 and try question 15 again.

As we have noted previously, any two numbers are either equal, or the first is larger than the second, or the first is smaller than the second. Put in terms of P and Q : either

$$P = Q$$

or $$P > Q$$

or $$P < Q$$

This makes exactly three relationships; only one of these is true for a specific P and Q . You should understand that unequal is not a separate relationship, but rather a combination of larger than and smaller than. Therefore, your choice is a true statement. But weren't you asked for a false statement!

Return to page 162/2 and try this question again.

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We proceed as follows:

$$2(x + y) - 5x = 6(2x - 3y) \qquad [\ D \qquad \text{(Use the DISTRIBUTIVE PROPERTY)}$$

$$2x + 2y - 5x = 12x - 18y \qquad \text{(Combine like terms)}$$

$$2y - 3x = 12x - 18y \qquad \prec -2y \qquad \text{(Subtract } 2y \text{ from both sides)}$$

$$-3x = 12x - 20y$$

Then you derived equation agrees with this choice.

Please proceed to question 18 below.

Question 18

Apply your knowledge to find the true statement about the solution of the equation

$$5(2x + 1) - 2(x - 3) = 7(4 - x)$$

(A) $x < 0$

(B) $0 < x < 1$

(C) $1 < x < 2$

(D) $x > 2$

Solving this equation, we have:

$x - a = 2a + b$ $\quad \propto + a$ (Adding a to both sides of the equation)

$x = 3a + b$

Then this choice is correct.

Please proceed to question 10 below.

Question 10

Solve for y . Choose the solution of the equation

$$3m + y = 7 - p$$

(A) $y = 4 - p - m$

(B) $y = 7 - p - 3m$

(C) $y = 3m + p - 7$

(D) $y = 4m - p$

158/1

It appears that you forgot about the DISTRIBUTIVE PROPERTY. When you apply that property to the left side of the equation, what is the result? Now you should be able to locate your error.

Please return to page 167/2 and try question 16 again.

158/2

There are many possible sequences of steps that will produce a solution. Try this one:

[D , [C , ∝ -2w , ∝ +2 , ∝ ÷ (-3)

Then be careful in describing your answer.

Your choice was not correct.

Please return to page 166/2 and try the problem again.

Since the original equation has t multiplied by 2, it requires a division by 2 to remove the coefficient from the t.

Then we must divide both sides by 2 getting

$$t = \frac{p - 2w}{2}$$

Your choice is correct.

Of course, it would also be proper to complete the division if it were possible. This would involve the DISTRIBUTIVE PROPERTY; that is, both the p and the $-2w$ would have to be divided by 2.

Then

$$t = \frac{p}{2} - w$$

Please proceed to question 13 below.

Question 13

Apply your knowledge to solve the equation

$$A = p + prt$$

for r.

(A) $r = \frac{p - A}{pt}$

(B) $r = \frac{A - p}{pt}$

(C) $r = \frac{A}{t}$

(D) $r = \frac{A}{2pt}$

V

$\frac{160}{1}$

In the original equation, the quantity

$$(P - y)$$

was multiplied by the fraction

$$\frac{2}{3}$$

The DISTRIBUTIVE PROPERTY tells us that both terms must be multiplied by the fraction. Then the $-y$ was also multiplied by it. What process of mathematics did you use to remove the multiplier from the y ?

In any case, this choice is not correct.

Return to page $\frac{150}{2}$ and try question 15 again.

$\frac{160}{2}$

If you reverse the second inequality, so that the given information reads:

$$m > f$$

and

$$f > t$$

you should realize that this choice is not correct. Do you recognize the relation which tells you how to do this problem? Sometimes it helps your understanding by substituting numbers for the letters and examining the relationships that result.

Return to page $\frac{173}{2}$ and try question 3 again.

Working carefully step by step, we have:

$$5(2x+1) - 2(x-3) = 7(4-x) \quad [\ D \quad \text{(DISTRIBUTIVE PROPERTY)}$$

$$10x + 5 - 2x + 6 = 24 - 7x \quad [\ C\wedge A \quad \text{(Combining terms)}$$

$$8x + 11 = 28 - 7x \quad \swarrow +7x \quad \text{(Add } 7x \text{ to both sides)}$$

$$15x + 11 = 28 \quad \swarrow -11 \quad \text{(Subtract } 11 \text{ from both sides)}$$

$$15x = 17 \quad \swarrow \div 15 \quad \text{(Divide both sides by } 15\text{)}$$

$$x = \frac{17}{15}$$

Since this number is larger than 1 and smaller than 2 , this choice is correct.

Please proceed to question 19 below.

Question 19

Apply your knowledge to solve the equation

$$\frac{2}{3}(n-6) = 5n + 9$$

(A) $n \in \{\text{positive odd integers}\}$

(B) $n \in \{\text{positive fractions}\}$

(C) $n \in \{\text{negative odd integers}\}$

(D) $n \in \{\text{negative fractions}\}$

V

162
1

Directed numbers can be pictured as points on a line, but two points are either located together or one is to the left of the other. In the same way, two numbers are either equal, or the first is larger than the second, or the first is smaller than the second.

It is impossible for two of these relationships to exist at the same time. Then this choice is correct since none of the other choices are.

Please proceed to question 2 below.

162
2

Question 2

If points P and Q are somewhere on the NUMBER LINE, which set of statements would you recognize as FALSE:

Choose the letter next to the false set.

(A) $p \neq Q$

(B) $P < Q$ or $P > Q$ (if P and Q are different)

(C) $P = Q$ or $P \neq Q$

(D) $P < Q$ or $P = Q$ or $P > Q$

In order to solve this equation, we proceed as follows:

$3(x + 1) = 5 - (1 - x)$	[D	(Use the DISTRIBUTIVE PROPERTY)
$3x + 3 = 5 - 1 + x$	[C^A	(Combine like terms.)
$3x + 3 = 4 + x$	<-x	(Subtract x from both sides)
$2x + 3 = 4$	<-3	(Subtract 3 from both sides)
$2x = 1$	<÷2	(Divide both sides by 2)
$x = \frac{1}{2}$		

Then this choice is correct.

Note that $-(1 - x)$ can be considered as

$$-1(1 - x)$$

That may make it easier to follow the second step above.

Please proceed to question 17 below.

Question 17

Apply your knowledge to find the derived equation which is equivalent to

$$2(x + y) - 5x = 6(2x - 3y)$$

(A) $-3x = 12x - 16y$

(B) $-3x = 12x - 20y$

(C) $2x + 2y = 7x - 18y$

(D) $2y = 9x - 18y$

Proceeding step by step, we have:

$A = p + prt$ $\quad \propto -p$ (Subtract p from both sides)

$A - p = prt$ $\quad \propto \div pt$ (Divide both sides by pt)

$\frac{A - p}{pt} = r$ [Using the SYMMETRIC PROPERTY]

$r = \frac{A - p}{pt}$

Therefore, this choice is correct.

Note that if the DISTRIBUTIVE PROPERTY was applied, you would get:

$r = \frac{A}{pt} - \frac{p}{pt}$

$r = \frac{A}{pt} - \frac{1}{t}$ which would be an alternate form, but one that is not preferred.

Please proceed to question 14 below.

Question 14

Apply the principles of operations on equations to solve the equation

$$n = \frac{p - k}{5k}$$

for p .

(A) $p = 6kn$

(B) $p = 5kn + k$

(C) $p = 5n$

(D) $p = 5n - 1$

It may be true that we cannot determine the value of each of the letters, but the relation between the letters m and t can be determined.
If necessary, try writing the second inequality in reverse order, and then examine your notes.

Please return to page 173/2 and try question 3 again.

If we use the SUBTRACTION PROPERTY for inequalities, and subtract 5 from each side, we find that this choice is not correct.
Did you confuse this with the multiplication or division rule?

$$n + 5 < 3 \qquad \propto -5$$
$$n < -2$$

Note: the symbol $\propto$ means to perform the same operation on both sides of the inequality.

Please return to page 175/2 and try question 6 again.

V

166
1

There are many sequences of steps resulting in a solution.

Let's examine two of them.

$$\frac{2}{3}(n-6) = 5n+9 \quad \times 3$$
$$2(n-6) = 3(5n+9) \quad [\text{D}$$
$$2n-12 = 15n+27 \quad \times -2n$$
$$-12 = 13n+27 \quad \times -27$$
$$-39 = 13n \quad \times \div 13$$
$$-3 = n$$

$$\frac{2}{3}(n-6) = 5n+9 \quad [\text{D}$$
$$\frac{2}{3}n-4 = 5n+9 \quad \times +4$$
$$\frac{2}{3}n = 5n+13 \quad \times -5n$$
$$\frac{2}{3}n-5n = 13 \quad \left(5 = \frac{15}{3}\right)$$
$$\frac{2n}{3}-\frac{15n}{3} = 13 \quad (\text{Collect})$$
$$\frac{-13n}{3} = 13 \quad \times 3$$
$$-13n = 39 \quad \times \div (-13)$$
$$n = -3$$

The value of n is a negative odd integer.

Please proceed to question 20 below.

166
2

Apply your knowledge to solve the equation

$$5w - 2(3w+1) = 2(w-7)$$

Which statement about the value of w is correct?

(A) $3 \leq w \leq 4$

(B) $4 < w \leq 5$

(C) $w > 5$

(D) $w < 3$

Solving this equation can be done in various ways.

One way is:

$$T = \frac{2}{3}(P - y) \quad \text{(multiply both sides by } \frac{3}{2}\text{)}$$

$$\frac{3}{2}T = P - y \quad \text{(add } y \text{ to both sides)}$$

$$y + \frac{3}{2}T = P \quad \text{(subtract } \frac{3}{2}T \text{ from both sides)}$$

$$y = P - \frac{3}{2}T$$

Then this choice is correct.

Please proceed to question 16 below.

Question 16

Apply your knowledge to solve the equation

$$3(x + 1) = 5 - (1 - x)$$

(A) $x = \frac{1}{2}$

(B) $x = 4$

(C) $x = \frac{1}{4}$

(D) $x = \frac{3}{2}$

VOLUME 5 SEGMENT 5 BEGINS HERE :

Obtain a PUNCH CARD from your instructor. In addition to the other identifying information that must be furnished by you, you are asked to punch out the following:

COLUMNS	48	and	50	2 5	(Sequence Number)
	54	and	56	0 4	(Type of Punch Card)
	60	and	62	0 5	(Volume Number)
	66	and	68	0 5	(Segment Number)

Your READING ASSIGNMENT for this Segment is pages 159 - 162 .

SUPPLEMENTARY NOTES:

There are two ways in which work with inequalities differs from work with equations.

(1) The symmetric property does NOT apply to inequalities, although it does for equations. That is, if $p = q$, then $q = p$; but if $p > q$ it is NOT true that $q > p$. This can be remembered by noticing that an inequality has a certain ORDER.

(2) Any multiplier or divisor (except zero) may be used on both sides of an equation and the result will be an equivalent equation. For an inequality, the result depends on whether the multiplier or divisor is positive or negative. It may help if you memorize the fact that a negative multiplier or divisor changes the order of the inequality.

You should carefully copy into your NOTEBOOK the rules listed in the pink bands on pages 159 , 160 , and 161 of your textbook.

You will now be asked a series of questions to draw your attention to the more important points.

Question 1

If p and q are two specific directed numbers, which choice do you recognize as a possible relationship between them with respect to size?

(A) $p > q$ and $p = q$

(B) $p > q$ and $p < q$

(C) $p < q$ and $p = q$

(D) None of these.

How did you decide on your answer? A simple numerical illustration will show that this choice is not correct.

Suppose

$$n = 6$$

$$w = 5$$

and $$h = 7$$

Then it is true that

$$n > w$$

since $$6 > 5$$

It is also true that

$$w < h$$

since $$5 < 7$$

But it is not true that

$$n > h$$

since $$6 < 7$$

Please return to page 178/2 and try question 4 again.

If we substitute

$$n = -5$$

in the original inequality, we have:

Given: $$n + 5 < 3$$

$$-5 + 5 < 3$$

$$0 < 3$$

which is certainly true. But this is not the only value of n which will check. Therefore, this choice is not correct. It is over-restrictive.

Please return to page 175/2 and try question 6 again.

V

170
1

If you will re-read your notes carefully, you will find that this problem is covered directly by one of the rules you were asked to learn.

This choice is not correct.

Why don't you try the technique of trying different arithmetic examples and interpreting them. Such examples will help you understand the rule and, therefore, learn it.

Please return to page 188/2 and try question 5 again.

170
2

In order to remove the -10 from the left side of the inequality, it was necessary to add 10 to both sides. But why did you change the order of the inequality? The order never changes when you add a quantity to both sides (either positive or negative).

Please return to page 194/2 and try question 8 again.

Applying the procedures you have learned, you get:

$$5w - 2(3w + 1) = 2(w - 7) \qquad [\ D$$

$$5w - 6w - 2 = 2w - 14 \qquad [\ \text{Collect}$$

$$-w - 2 = 2w - 14 \qquad \swarrow -2w$$

$$-3w - 2 = -14 \qquad \swarrow +2$$

$$-3w = -12 \qquad \swarrow \div(-3)$$

$$w = 4$$

Since this choice includes all values from 3 to 4 (including 4) , this choice is correct.

You have now finished this Segment.

Hand in the PUNCH CARD.

You should now be able to complete ASSIGNMENT 5 Problems, 13-16 .

V

172/1

What method did you use to arrive at this answer?

The statement

$$w < h$$

can be changed to $h > w$

Then the original problem means that

$$n > w$$

and $h > w$

If two numbers are both greater than another number, can you say definitely that they are equal? They might be equal, but could you be sure?

This choice is not correct.

Please return to page 178/2 and try question 4 again.

172/2

Given:

$$x + 3 > -7 \qquad \not< -3$$

$$x > -10$$

If we subtract 3 from both sides of the inequality, we get

$$x > -10$$

How did you get the order of the inequality reversed?

This choice is not correct.

Please return to page 184/2 and try question 7 again.

The fact that P and Q were mentioned as being on the NUMBER LINE does not imply that the points are actually different points. There is still the possibility that they are the same; that is,

$$P = Q$$

Therefore, although there are many possibilities that

$$P \neq Q$$

it is not a true statement.

Your choice of a false statement is correct.

Please go on to question 3 below.

Question 3

There are three directed numbers:

$$m \ , \ f \ , \text{ and } t$$

If $m > f$

and $t < f$

choose the relation between m and t .

( ) $m > t$

(B) $m = t$

(C) $m < t$

(D) Cannot be determined.

V

$\frac{174}{1}$

The order of the inequality in this choice is different from the order in the original problem. The order changes only when you multiply or divide by a NEGATIVE number on both sides of the inequality.

What operation did you perform?

This choice is not correct.

Please return to page $\frac{194}{2}$ and try question 8 again.

$\frac{174}{2}$

This choice is not correct.

There is a specific rule about a multiplication which changes the order of an inequality.

if $a < b$ and $c < 0$

then $ac > bc$

Note: the order is reversed. Apply this rule to this question.

Please return to page $\frac{191}{2}$ and try question 10 again.

This is an illustration of the additive property of inequality, which is true for any number added to both sides of an inequality.

It is similar to the additive property of equality which states:

if $x = y$ then, $x + z = y + z$

Then this choice is correct.

Please proceed to question 6 below.

Question 6

If it is given that

$$n + 5 < 3$$

choose the correct conclusion concerning the value of n.

(A) $n > -2$

(B) $n < -2$

(C) $n = -5$

(D) $n = -6$

V

176/1

The rule for multiplication in inequalities states that multiplying by a negative number reverses the order of the inequality.

You started with an inequality. If this inequality was transformed by multiplying both sides by -1, you would still end up with an inequality with the order reversed.

You were asked to transform the inequality into an equality, but the statement

$$mw = nw$$

is not an inequality. Therefore, this choice is not correct.

Please return to page 186/2 and try question 9 again.

176/2

Apparently you have performed a division on both sides of the inequality. However, you appear to have made a mistake in using the rule for division since this choice is not correct.

Note: the operation

$$\not\div(-k)$$

changes the order of the inequality.

Please return to page 181/2 and try question 11 again.

It appears that you have subtracted 3 from the left side of the inequality. But subtracting 3 from -7 does not give -4 .

Given:

$$x + 3 > -7 \quad \not< -3$$

$$x > -10$$

This choice is not correct.

Please return to page 184/2 and try question 7 again.

The DISTRIBUTIVE PROPERTY can be applied whenever the expression

$$a (b + c)$$

appears. Here it is part of an inequality.

The first step necessary is to apply the DISTRIBUTIVE PROPERTY to simplify the left side of the inequality. It is then necessary to perform an addition and finally a division on both sides of the inequality.

Have you forgotten that an inequality can be checked in a way similar to checking an equation? Substitute your answer choice in the original relationship.

Try

$$n = 3$$

Since you can discover this result is not correct, where did you make your mistake?

Please return to page 195/2 and try question 13 again.

V

If we reverse the writing of the second inequality, we have

$$m > f \quad \text{and} \quad f > t$$

The transitive relation of inequalities can be expressed in words as follows:

> "If the first of three quantities is larger than the second, and the second is larger than the third, then the first is larger than the third."

Then the transitive relation of inequality tells us that

$$m > t$$

so that this choice is correct.

Please proceed to question 4 below.

Question 4

If

$$n > w \quad \text{and} \quad w < h$$

choose the relation between n and h.

(A) $n > h$

(B) $n = h$

(C) $n < h$

(D) Cannot be determined.

If we substitute

$$n = -6$$

in the original inequality, we have:

Given: $n + 5 < 3$

$-6 + 5 < 3$

$-1 < 3$

which is certainly true. But this is not the only value of n which will check. Therefore, this choice is not correct. It is too restrictive.

Please return to page 175/2 and try question 6 again.

In order to transform the left side of the given inequality into the left side of the one in this choice, we subtract 6 from both sides and then double both sides as follows:

Given: $3x + 6 > -9$ $\not{-6}$

$3x > -15$ $\not{\cdot 2}$

$6x > -30$

However, you have made a mistake if you got this choice as a result. Where did you make the error?

Please return to page 187/2 and try question 12 again.

V

180
1

It is quite possible that you would not meet any of the other choices if you attempted to solve the inequality. However, the question asked only for a statement which was correct when the given inequality is known. You will discover that one of the other choices is the result of a proper application of one of the rules for transforming inequalities.

One technique is to find some numerical value for the variable in the original inequality making it true. This same value then can be substituted in each choice to check its truth value.

Please return to page 194/2 and try question 8 again.

180
2

Applying the proper procedure for solving the inequality, we have:

$$\begin{aligned} 2(3 - y) - 7 &\leq 13 && [\ D \\ 6 - 2y - 7 &\leq 13 && [\ C \wedge A \\ -2y - 1 &\leq 13 && \not< + 1 \\ -2y &\leq 14 && \not< \div (-2) \\ y &\geq -7 \end{aligned}$$

Since our divisor is negative, the order of the inequality changes. Then this choice is not correct since it does not agree with the solution set given.

Please return to page 199/2 and try question 20 again.

There is a rule about a multiplication which changes the order of an inequality. It requires that the multiplier by a negative number. This rule can be expressed in symbolic form.

if	$a > b$	and	$c < 0$
then	$ac < bc$		

Then none of the other choices is correct, and this one is.

Please proceed to question 11 below.

Question 11

Apply your knowledge of operations on inequalities to determine the value of x when you are given:

$$-3x < -12$$

Choose the correct statement about the value of x.

(A) $x < 4$

(B) $x < -4$

(C) $x > 4$

(D) $x > -4$

182
1

If you will review your notes concerning the multiplication of both sides of an inequality by a number, you will discover that there is a value that makes this conclusion true. However, this choice is not correct.

Please return to page 186/2 and try question 9 again.

182
2

This choice says that there is no solution to the inequality, but that is not correct since it is obvious that the number 1 will check.

We would have

$$5 - 3(1) < 1 + 5$$

or $$5 - 3 < 6$$

or $$2 < 6$$

and this is certainly true. Perhaps you didn't go through all the steps necessary to find the solution of an inequality.

Please return to page 198/2 and try question 15 again.

V

Are you sure of this answer? To help our understanding let us try an example of the general case.

Suppose

$n = 6$

$w = 2$

and $h = 3$

Then it is true that $n > w$

since $6 > 2$

It is also true that $w < h$

since $2 < 3$

But is not true that $n < h$

since $6 > 3$

Therefore, this choice is not correct.

Return to page 178/2 and try question 4 again.

We start with the given inequality and choose an operation that will transform the left side into the left side of your choice.

If we subtract 8 from both sides of the inequality, we will obtain an equivalent inequality.

Given:

$3x + 6 > -9$ $\not< -8$

$3x - 2 > -17$

Subtracting 8 from the left side gives

$3x - 2$

but subtracting 8 on the right side does not give us the value offered in this choice. Then this choice is not correct.

Return to page 187/2 and try question 12 again.

V

$$n + 5 < 3 \quad \text{⋉} -5$$

$$n < -2$$

Subtracting 5 from each side of the inequality transforms this into an equivalent inequality, and we get

$$n < -2$$

Then this choice is correct. The symbol ⋉ means perform that operation on both sides of the inequality.

Note that the order of the inequality changes ONLY when you multiply or divide on both sides by a negative number.

Please proceed to question 7 below.

Question 7

If you are given that

$$x + 3 > -7$$

what can be said about the value of x ?

Choose the letter next to the conclusion.

(A) $x < -10$

(B) $x < -4$

(C) $2x + 6 > -14$

(D) $2x + 6 < -14$

The DISTRIBUTIVE PROPERTY can be applied whenever the expression

$$a (b + c)$$

appears. Here it is part of an inequality.

The first step necessary is to apply the DISTRIBUTIVE PROPERTY to simplify the left side of the inequality. It is then necessary to transform the inequality by performing addition and then division on both sides.

What is it that caused the order of the inequality to reverse. The only condition under which an inequality reverses order is when it is multiplied (or divided) on both sides by a negative number.

Since this choice is not correct, you have made at least one mistake somewhere in this process.

Please return to page 195/2 and try question 13 again.

The successive steps in this problem involve the DISTRIBUTIVE PROPERTY, the SUBTRACTION PROPERTY of inequalities, and the DIVISION PROPERTY of inequalities.

Unless each step is perfect the result is incorrect. How many mistakes did you make, and where were they?

Please return to page 202/2 and try question 16 again.

V

186
1

If we divide both sides of the inequality by +5 , the order of the inequality does not change, and we get

$$n - 2 > -3$$

i. e.

$$5n - 10 > -15 \quad \div 5$$

$$\frac{1}{5}(5n - 10) > -3 \quad [\text{ D}$$

$$n - 2 > -3$$

Then this choice is correct.

Please proceed to question 9 below.

186
2

Question 9

If

$$m > n$$

choose the value of w

for which $mw = nw$

(A) 0

(B) -1

(C) No value

(D) Some value not listed.

V

Let's review the problem:

Given:

$$-3x < -12 \qquad \not\times \div(-3)$$

$$x > +4 \qquad \text{(order reversed)}$$

In order to remove the -3 attached to the x by multiplication, we must divide both sides of the inequality by -3.

Then the order of the inequality also changes. Watching our rules for dividing signed numbers, we then get

$$x > 4$$

and this choice is correct.

Please proceed to question 12 below.

Question 12

Apply your knowledge of operations on inequalities to find the one correct statement listed below that can be derived from

$$3x + 6 > -9$$

(A) $6x < -30$

(B) $-6x < +30$

(C) $3x - 2 > -13$

(D) $3x - 2 > -1$

V

By reversing the order of the second inequality, we have

$$n > w \quad \text{and} \quad h > w$$

Then since both n and h are larger than w, the question is which one of these is larger. If both n and h are 2 larger than w they would be equal. If n were larger by 3 while h were larger by 2 then n would be larger than h.

But the situation could just as well be reversed. That is, it is impossible to tell from the given information, how the sizes of n and h compare.

Please proceed to question 4 below.

Question 5

State which choice is correct about the conclusion that

if $x > y$

then $x + z > y \div z$

(A) It is always true

(B) It is never true

(C) It may be true or not, depending on the value of z.

(D) It may be true or not, but the value of x does not matter.

V

It appears that you have multiplied both sides of the inequality by the same number, which is correct. But why did you change the order of the inequality? There is a rule telling you when the order of the inequality is to be changed, but you have not used it correctly. The inequality sign changes order when the operation is either multiplication or division by a negative value.

if $a > b$ $\times(-k)$

then $-ak < -bk$

or

if $a > b$ $\times \div (-k)$

then $-\frac{a}{k} < -\frac{b}{k}$

Return to page 184/2 and try question 7 again.

The steps is the solution of this inequality are two different subtractions from both sides. It is important that you make no mistake in handling the signed numbers, or the fractions; you must also be careful to follow every rule you have learned for handling inequalitites.

Somewhere you have an error since this choice is not correct.

Return to page 196/2 and try question 17 again.

V

In solving this inequality, we proceed as follows:

$$\begin{aligned} 3(2n - 6) &> 6 && [\text{D} \\ 6n - 18 &> 6 && \cancel{+}\ 18 \\ 6n &> 24 && \cancel{\div}\ 6 \\ n &> 4 \end{aligned}$$

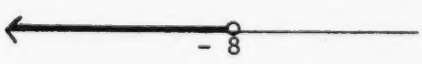
Note that the order of the inequality does not change since the divisor was positive.

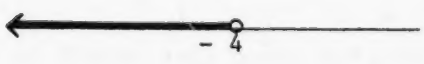
Please proceed to question 14 below.

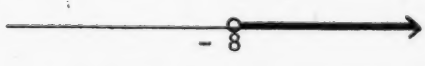
Question 14

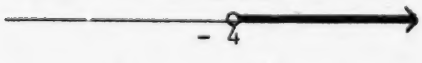
Apply your knowledge to find the graph of the solution set of the inequality

$$2(5 - y) < 18$$

(A) 

(B) 

(C) 

(D) 

The only multiplier which can turn an inequality into an equation is the number zero.

Given:

$$m > n \qquad \times 0$$

$$0 \cdot m \;?\; 0 \cdot n$$

$$0 = 0$$

Therefore, this choice is correct.

Please proceed to question 10 below.

Question 10

Given that

$$p > q \quad \text{and} \quad mp < mq$$

Choose the correct statement about the value of m .

(A) m is positive

(B) m is zero

(C) m might be negative or positive

(D) None of these.

192
1

There were many opportunities for you to make a mistake in solving this inequality. It appears that you were successful in avoiding errors since this choice is correct. The work could have been done as follows:

$$
\begin{aligned}
3(x-5) - 2(3x-1) &\geq 2 && [\text{ D} \\
3x - 15 - 6x + 2 &\geq 2 && [\text{ C}\wedge\text{A} \\
-3x - 13 &\geq 2 && \text{add } 13 \\
-3x &\geq 15 && \text{divide by } (-3) \\
x &\leq -5 && (\text{reverse order of inequality})
\end{aligned}
$$

Please proceed to question 19 below.

192
2

Question 19

Apply your knowledge to find the solution set of the inequality

$$8x + 5 - (2x + 1) < -8$$

(A) $x < -2$

(B) $x > \frac{3}{2}$

(C) $x < -\frac{7}{3}$

(D) $x > \frac{9}{2}$

The solution of this inequality involves subtractions to remove the x term from the right side and to remove the term 5 from the left side. It is then necessary to divide to get x alone. In which of these steps did you make your mistake?

This choice is not correct.

Return to page 198/2 and try question 15 again.

Applying the proper procedures to solve this inequality, we have:

$$5y + 3 \geq -32 \qquad \text{subtract } 3$$

$$5y \geq -35 \qquad \text{divide by } 5$$

$$y \geq -7$$

Since we divided by a positive number, the order of the inequality remains the same. Then this choice does not agree with the solution set given.

Return to page 199/2 and try question 20 again.

194
1

If we multiply both sides of the inequality by + 2 we get

$$2x + 6 > -14$$

Since our multiplier was positive, the order of the inequality remains unchanged.

Given:

$$x + 3 > -7 \qquad \times 2$$

$$2(x + 3) > 2(-7) \qquad [\text{D}$$

$$2x + 6 > -14$$

Then this choice is correct.

Please proceed to question 8 below.

194
2 Question 8

Apply your knowledge of operations on inequalities to find the proper derived inequality equivalent to:

$$5n - 10 > -15$$

Choose the letter next to the correct statement.

(A) $5n < -5$

(B) $n - 2 > -3$

(C) $n - 2 < -3$

(D) None of these.

In order to transform the left side of the given inequality into the left side of this choice we preform the following operations:

$$3x + 6 > -9 \qquad \times -6$$

$$3x > -15 \qquad \times (-2)$$

$$-6x < +30$$

Note: the order of the inequality is reversed.

Since the operations must be applied to both sides of the inequality, the right hand side was also transformed making the resulting inequality the same as your choice.

Please proceed to question 13 below.

Question 13

Apply your knowledge of operations on inequalities to find the solution set of the inequality:

$$3(2n - 6) > 6$$

(A) $n > 2$ (C) $n > 4$

(B) $n < 2$ (D) $n < 4$

196
1

Very good. This choice is correct.

The solution should look like this:

$$5m + 2(1 - 3m) \geq 10 \qquad [\ D$$

$$5m + 2 - 6m \geq 10 \qquad [\ C \wedge A$$

$$2 - m \geq 10 \qquad \times -2$$

$$-m \geq 8 \qquad \times \cdot(-1)$$

$$m \leq -8 \qquad \text{(Change the order of the inequality.)}$$

Please proceed to question 17 below.

196
2

Question 17

Apply your knowledge of inequality solution techniques to find the solution set of the inequality

$$2m + 7 > \frac{1}{2}m - 2$$

(A) $m < -6$

(B) $m > -6$

(C) $m > 3$

(D) $m > 0$

There were many opportunities for you to make a mistake.

You might look for the mistake in your use of the DISTRIBUTIVE PROPERTY at the beginning of the work, particularly in the case of

$$-2(3x - 1)$$

which equals

$$-6x + 2$$

The sign of the 2 is a typical trouble spot.

Please return to page 200/2 and try question 18 again.

There is one part of this problem which appears to have given you difficulty. You should consider

$$-(2x + 1)$$

as if it were written $-1(2x + 1)$

The two expressions have the same value, but the second form makes the application of the DISTRIBUTIVE PROPERTY easier to perform.

Secondly, the original inequality and your solution set have reversed orders of inequality. That condition could only have occured if you multiplied or divided by a negative number. You shouldn't have had to do that if you proceeded properly.

Be careful to apply specific rules in each step of your work.

Please return to page 192/2 and try question 19 again.

198
1

You have done very well. There are many steps which must be done correctly in order to get this correct choice. We should proceed as follows:

$$2(5 - y) < 18 \qquad [\ D$$

$$10 - 2y < 18 \qquad \not< -10$$

$$-2y < 8 \qquad \not< \div (-2)$$

$$y > -4$$

Of course, we must be careful to change the order of the inequality since we divided by a negative number.

Since we want all those values greater than -4, our graph must include all points to the right of -4, and not -4.

Therefore, the -4 is circled on the graph.

Please proceed to question 15 below.

198
2

Question 15

Apply the appropriate principles of operations on inequalities and choose the correct statement about the solution set of the inequality

$$5 - 3x < x + 5$$

(A) $x > -2.5$ (C) $x > 0$

(B) $x > -5$ (D) $x \in \emptyset$

Very good. This choice is correct. Your work should look like this:

$8x + 5 - (2x + 1) < -8$ [D Note:

$8x + 5 - 2x - 1 < -8$ [C ^ A $-(2x + 1) = -1(2x + 1)$

$6x + 4 < -8$ $\times -4$

$6x < -12$ $\times \div 6$

$x < -2$

Please proceed to question 20 below.

Question 20

Apply your knowledge to find the inequality whose solution set is

$$y \leq -7$$

(A) $5y + 3 \geq -32$

(B) $-7(y + 2) + 3(y - 5) \geq -1$

(C) $3(y + 6) + 2 \geq -1$

(D) $2(3 - y) - 7 \leq 13$

Applying the proper procedures, we have:

$2m + 7 > \frac{1}{2}m - 2$ (Change 2 to a fraction)

$\frac{4}{2}m + 7 > \frac{1}{2}m - 2$ $\not< -\frac{1}{2}m$

$\frac{3}{2}m + 7 > -2$ $\not< -7$

$\frac{3}{2}m > -9$ $\not< \cdot 2$

$3m > -18$ $\not< \div 3$

$m > -6$

Note that our divisor was positive, therefore, the inequality did not reverse in order.

Please proceed to question 18 below.

Question 18

Apply your knowledge to find the solution set of the inequality

$$3(x - 5) - 2(3x - 1) \geq 2$$

(A) $x \leq -\frac{19}{3}$ (C) $x \geq -5$

(B) $x \leq -6$ (D) $x \leq -5$

By this time you should realize that the technique of solving inequalities is almost identical with those methods you have already learned in solving equations.

There is one exception; when you multiply or divide by a negative number the order of the inequality changes.

You can always check your answer by choosing a member of the solution set and substituting that. It is not a perfect check, however. But if it doesn't check, you know you are wrong; whereas if it does check, it doesn't mean that you are right unless you have considered the limits of the possibilities.

Please return to page 192/2 and try question 19 again.

Applying the proper procedure for solving the inequality, we have:

$3(y + 6) + 2 \geq -1$	[D
$3y + 18 + 2 \geq -1$	[C ^ A
$3y + 20 \geq -1$	⋉ - 20
$3y \geq -21$	⋉ ÷ 3
$y \geq -7$	

Since our divisor is positive, the order of the inequality does not change. Then this choice is not correct, since it does not agree with the solution set given.

Return to page 199/2 and try question 20 again.

V

202
1

The work in this problem should be as follows:

$$5 - 3x < x + 5 \quad \text{✗} - x$$

$$5 - 4x < 5 \quad \text{✗} - 5$$

$$-4x < 0 \quad \text{✗} \div (-4)$$

$$x > \frac{0}{-4}$$

Remember to change the order of the inequality.

$$x > 0$$

Please proceed to question 16 below.

202
2

Question 16

Apply your knowledge to find the solution set of the inequality

$$5m + 2(1 - 3m) \geq 10$$

(A) $m \leq -8$

(B) $m \geq -8$

(C) $m \geq 4$

(D) $m \leq 4$

It is important to realize that you cannot find the solution to an inequality by trying offered choices. For example,

$$m = 10$$

is a value which fits this choice. Substituting $m = 10$ in the inequality, we have

$$20 + 7 > 5 - 2$$

or

$$27 > 3$$

which is certainly true. But even if we tried every number which fits this choice (and there are an infinite number of them) , we would still need to show that no number which does not fit the choice would check in the inequality. Since this is impossible, we must follow procedures as taught in this segment if we wish to solve inequalities.

In any event, this choice is not correct.

Please return to page $\frac{196}{2}$ and try question 17 again.

There were many opportunities for you to make a mistake in solving this inequality. It appears that you were successful in avoiding errors until you were almost at the end of the solution. But you did have an error; therefore, this choice is not correct.

Remember, the symbol $\geq$ is an inequality and as such is reversed if the operation is multiplication or division by a negative number.

Please return to page $\frac{200}{2}$ and try question 18 again.

204
1

The procedure for this problem involves the DISTRIBUTIVE PROPERTY, subtracting a quantity from both sides, and finally dividing both sides by the same number. You must then translate your result into a graph showing where your solution would be located on the NUMBER LINE. It takes only one error in one of these steps to get a wrong answer. How many mistakes did you make to get this incorrect choice?

Please return to page 190/2 and try question 14 again.

204
2

Applying the proper procedures to solve the inequality, we have:

$$-7(y+2)+3(y-5) \geq -1 \qquad [\text{ D}$$

$$-7y-14+3y-15 \geq -1 \qquad [\text{ C} \wedge \text{A}$$

$$-4y-29 \geq -1 \qquad \not< +29$$

$$-4y \geq 28 \qquad \not< \div(-4)$$

$$y \leq -7 \qquad (\text{change the order of inequality})$$

Then this choice is correct.

You have now finished Segment 5 . Hand in the PUNCH CARD.
Be sure you have entered in your NOTEBOOK the rules for inequalities which you found in your text on pages 159 - 161 .

You should now complete ASSIGNMENT 5 , problems 17 - 20 . After completing the assignment, you are advised to review all of your notes and to re-read the reading assignment in order to prepare yourself for the Volume test.

V